

EFFICIENCY WITH ENDOGENOUS POPULATION GROWTH

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In this paper, we generalize the notion of Pareto efficiency to make it applicable to environments with endogenous populations. Two efficiency concepts are proposed: \mathcal{P} -efficiency and \mathcal{A} -efficiency. The two concepts differ in how they treat potential agents that are not born. We show that these concepts are closely related to the notion of Pareto efficiency when fertility is exogenous. We prove a version of the first welfare theorem for Barro–Becker type fertility choice models and discuss how this result can be generalized. Finally, we study examples of equilibrium settings in which fertility decisions are not efficient, and we classify them into settings where inefficiencies arise inside the family and settings where they arise across families.

KEYWORDS: Pareto optimality, first welfare theorem, fertility, dynasty, altruism.

1. INTRODUCTION

INTEREST IN THE DETERMINANTS of the equilibrium path for population has increased recently. (See Becker and Barro (1988), Barro and Becker (1989), Raut (1991), Galor and Weil (1996, 2000), Doepke (2004), Fernandez-Villaverde (2001), Boldrin and Jones (2002), and Tertilt (2005); see Nerlove and Raut (1997) for a survey.) Surprisingly, little of this literature has used the tools of modern welfare economics (for example, Debreu (1959)) to address the normative questions that arise. This is because, at least in part, the usual notion of Pareto efficiency is not well defined for environments in which the population is endogenous. To illustrate this, consider the following example. Compare an allocation with two agents, each consuming one unit of a lone consumption good, with an allocation where only one agent is born, but consumes two units of the consumption good. Is one allocation Pareto superior to the other? Pareto efficiency would involve a comparison, for each person, of the two allocations. However, because different sets of people are alive in the two allocations, such a person-by-person comparison seems impossible.

In this paper, we generalize the notion of Pareto efficiency to make it applicable to environments with endogenous populations. We propose two new efficiency concepts: \mathcal{P} -efficiency and \mathcal{A} -efficiency. They differ in the way that potential agents that are not born are treated. In \mathcal{P} -efficiency, unborn children are treated symmetrically with the born agents (i.e., they have utility functions, etc.), but with a limited choice set.² In \mathcal{A} -efficiency, efficiency is defined only

¹We would like to thank Kenneth Arrow, Doug Bernheim, Partha Dasgupta, Ed Prescott, Igor Livshits, and seminar participants at numerous locations for helpful discussions and comments, and the National Science Foundation (SES-0519324), the Federal Reserve Bank of Minneapolis, and the Stanford Institute for Economic Policy Research for financial support. We would also like to thank three anonymous referees and a co-editor for helpful comments.

²Throughout, we do not take a stand on how to evaluate the utility of the unborn. Such a task is well beyond the scope of this paper. Rather, we propose two alternative definitions of Pareto

through comparisons among agents that are born (and hence it is not necessary that the unborn have well defined utility functions). We show that these two concepts are closely related to the notion of Pareto efficiency when fertility is exogenous. We then discuss how these concepts are related to each other. We also give results with regard to the existence of efficient allocations and derive planning problems that partially characterize the set of efficient allocations.

To do this, we provide a fairly general formulation of fertility choice. Naturally, such a formulation will be embedded in an overlapping generations framework. Each decision maker has a fixed set of potential children and decides how many of them will be born. Models of fertility also naturally involve external effects across agents in the economy. We allow for any individual's utility to depend on the consumption of other family or dynasty members. This includes the Barro and Becker (1989) formulation of fertility along with many others. In addition to this utility externality, there is another more subtle one. From the point of view of the potential children, this is a model in which their choice set is dependent on the actions of other agents in the economy. If the parent chooses that they will not be born, they have effectively no choices.

As is usual in models with external effects, there is no presumption that individual behavior will aggregate to an efficient outcome. There are some cases, in which they do, however. We show that in a model with Barro–Becker style preferences, external effects between family members do not cause a problem for efficiency because preferences of parents and their descendants are similar enough. The key insight here is that if dynasties act as if they were maximizing subject to an infinite horizon dynastic budget constraint, then equilibrium allocations will be efficient.

Our approach allows us to distinguish easily between two potential reasons for concern about overpopulation that have been at the center of the more recent debates on population. The first of these is the existence of scarce factors and the “crowding” of these factors that results when the population is large. The second concern is the potential increase in pollution (e.g., emission of greenhouse gases) as population grows. We show that scarce factors do not, in and of themselves, give rise to inefficiencies in population. Rather, this externality is “pecuniary” with effects manifested in price changes. (This is similar to the arguments made in Willis (1987) and Lee and Miller (1991).) In contrast, if true external effects exist that are related to population size, not surprisingly, individual choices do not necessarily lead to efficient population sizes. This is true both when the external effects are negative, like pollution, and when they are positive, for example, knowledge spillovers (Romer (1986)) or human capital externalities (Lucas (1988)).³ Of course, part of the debate

optimality that are at opposite extremes of the spectrum of treatments of the unborn. For either notion, a version of the first welfare theorem holds.

³Interestingly, Keynes was one of the first authors to argue that population growth was too low in England in the 1920s, and that this was a cause for a reduction in inventive activity and hence stagnation (see Zimmermann (1989)).

about overpopulation is a question of distribution of resources, that is, which of many efficient allocations is the best. Although our concepts have nothing to say about optimal redistribution among agents, we believe that identifying inefficiencies is an important first step toward such an even more ambitious goal.

The problem that Pareto efficiency is not well defined in the endogenous population context has been long recognized in the literature. The debate over alternative concepts dates back to at least Mill (1848) and Bentham (1823), who proposed per capita utility and the sum of utilities, respectively, as alternative welfare concepts.⁴ Early papers employed these alternative social welfare functions in the context of models where children do not affect preferences and parents do not choose fertility (e.g., Samuelson (1975) and Dasgupta (1969)). The more recent literature assumes that a parent's utility depends on his own consumption and on both the utility of and number of his children, and uses the Millian and Benthamite criteria to compare population sizes in equilibrium with the optimal one (Nerlove, Razin, and Sadka (1987, 1989), Razin and Sadka (1995)). Eckstein and Wolpin (1985) maximized utility of a representative agent instead. Such criteria, however, typically give one optimal allocation and are very different in spirit from an efficiency concept that usually contains a large number of allocations.

A small recent literature addresses the question of optimal populations using a Paretian approach. The works of Schweizer (1996) and Conde-Ruiz, Gimenez, and Perez-Nievas (2004) are most closely related to our approach. Each paper proposes a new efficiency concept and proves versions of the first and second welfare theorems. However, these papers propose concepts that are sufficiently less general than ours, are defined only for symmetric environments, and focus exclusively on allocations that are identical for all people within a generation. Michel and Wigniolle (2007) use a concept that compares utilities generation by generation. Within the context of a specific model, they give an example that shows that the concept of "golden rule" should be modified in the context of endogenous populations. Willis (1987) also attempts to analyze whether general equilibrium models with endogenous fertility lead to Pareto efficient allocations. Willis does this, however, without formally defining Pareto efficiency for these environments. Instead, Willis studies the solution to a planning problem and shows the conditions under which it coincides with a competitive equilibrium.

An alternative approach is that from the social choice literature. There, authors use an axiomatic approach to derive representation theorems for social orderings that include population size as one of the choices (see, for example, Blackorby, Bossert, and Donaldson (1995), Broome (2003, 2004)).⁵ These representation theorems have a particularly simple and intuitive form known

⁴See Zimmermann (1989) for an excellent summary of the historic debate.

⁵Section 6 of Blackorby, Bossert, and Donaldson (2002) provides an excellent survey.

as critical level utilitarianism—a new person should always be added to the population as long as the value to society of doing this exceeds some critical level. As with the Millian and Benthamite criteria, the goal of this literature is to determine one optimal population size. Our approach is different and complementary in that it gives definitions that are analogous to the usual Pareto frontier. As is typically the case even without the issues of endogenous fertility, this gives a large set of efficient outcomes while the social choice approach typically gives only one (for each critical level). On the plus side, our approach requires only ordinal comparisons and, hence, no judgements about the meaning of interpersonal comparisons of utility or issues about “scaling” utility functions is necessary. In addition, our approach naturally lends itself to addressing questions concerning the efficiency of privately chosen fertility levels without adding in the extra issues inherent to distributional questions.

Finally, a few authors have pointed out various reasons why the private and social costs of having children could differ (Friedman (1972), Chomitz and Birdsall (1991), Lee and Miller (1991), Simon (1992), and Starrett (1993)). These papers informally discuss types of externalities that could arise in the context of fertility choice, but none provides a formal concept or the tools to address the efficiency question thoroughly.

The remainder of our paper is organized as follows. In Section 2, we introduce notation. In Section 3, we give definitions of our two notions of Pareto efficiency, and discuss several properties of the concepts. Section 4 discusses the circumstances under which equilibrium allocations will be efficient. In Section 5, we apply the concepts to analyze several (potential) reasons for over- and underpopulation. Section 6 concludes.

2. NOTATION AND FEASIBLE ALLOCATIONS

Dasgupta (1995, p. 1899) pointed out that “developing the welfare economics of population policies has proved to be extremely difficult: our ethical intuition at best extends to actual and future people, we do not yet possess a good moral vocabulary for including potential people in the calculus.” In this section, we aim to make progress on this dimension by providing a new framework that makes extending the tools of modern welfare economics to questions of optimal populations possible. An important component of our framework is an explicit dynastic structure, something that has been largely ignored in the literature. The advantages of an explicit dynastic structure are threefold. First, it allows for external effects (e.g., altruism) between family members. It follows that even if the planner puts zero weight on a person, it might still be optimal for that person to be born, because a parent wants the child. That is, in our framework we take people’s preferences about other people explicitly into account. Second, we make it explicit that creating another person is costly and that this cost might not always be transferable (e.g., the time cost of a mother nursing a baby). Third, it introduces a natural asymmetry between people who

are alive for sure (the initial generation) and those who might or might not be born (everyone else).

Consider an overlapping generations economy, where each generation makes decisions about fertility. For simplicity, each agent is assumed to live only for one period. The initial population in period 0 is denoted by $\mathcal{P}_0 = \{1, \dots, N\}$. Each person can give birth to a maximum of \bar{f} children.⁶ For each period t , the potential population \mathcal{P}_t is defined recursively as $\mathcal{P}_t \equiv \mathcal{P}_{t-1} \times \mathcal{F}$, where $\mathcal{F} = \{1, \dots, \bar{f}\}$, and we denote by \mathcal{P} the population of all agents potentially alive at all dates. Simply put, \mathcal{P} is the set of all individuals that might be, depending on fertility choices, nodes of one of the N family trees, one for each time 0 agent. Then an individual born in period t is indexed by $i^t \in \mathcal{P}_t$ and can be written as $i^t = (i^{t-1}, i_t)$, specifying that i^t is the i_t th child of the parent i^{t-1} . For example, $i^t = (1, 3, 2)$ means that person i_t is the second child of the third child of person 1 $\in \mathcal{P}_0$. We often simply write i because the length of the vector already indicates the period in which the agent was born. Similarly, a fertility plan, denoted by f , is a description of the number of children born to each agent. Thus, $0 \leq f(i) \leq \bar{f}$ for all $i \in \mathcal{P}$. Each fertility plan f implicitly defines the subset (of \mathcal{P}) of individuals actually born under the plan f . This set will be denoted by $I(f)$ and is defined recursively first by, $i_0 \in I(f)$ for $i_0 \in \mathcal{P}_0$; then $(i_0, i_1) \in I(f)$ for $i_0 \in \mathcal{P}_0$ if and only if $i_1 \leq f(i_0)$; and so forth. Let $I_t(f) = I(f) \cap \mathcal{P}_t$ denote the set of people alive in period t under the fertility plan f . The set $I(f)$ is the set of N actual family trees realized under the fertility plan f , one for each time 0 agent or dynasty head. For $i_0 \in \mathcal{P}_0$, let D_{i_0} be the set of potential descendants of i_0 including i_0 himself; that is $i = (\hat{i}, i_1, \dots, i_t) \in D_{i_0} \Leftrightarrow \hat{i} = i_0$. Note that $D_i \cap D_{i'} = \emptyset$ if $i \neq i'$. We call D_i dynasty i . Then we can write $f = (f_{i_0})_{i_0 \in \mathcal{P}_0}$ when it is necessary to distinguish between the fertility plans for different dynasties. For any fertility plan f , we use the notation $I(f_i)$, $i \in \mathcal{P}_0$, to denote i and all of i 's descendants under the plan: $I(f_i) = I(f) \cap D_i$. Note that $I(f_i)$ does not depend on f_{-i} , but only on f_i . We denote the set of all fertility plans by F .⁷

We assume that there are k goods available in each period. Goods will be interpreted in a broad sense here; included are labor, leisure, and capital services. Given any fertility plan, a consumption plan, x , is a determination of the level of consumption of these k goods for each person that is actually born. That is, $x: I(f) \rightarrow \mathbb{R}^k$ where $x(i) \in \mathbb{R}^k$ represents the consumption of agent $i \in I(f)$. There is one representative firm, which behaves competitively. The

⁶Throughout most of the paper, we will assume that the number of children possible is discrete. Many of the models of fertility choice (e.g., Barro and Becker (1989)) allow for noninteger choices. Much of the analysis presented here can be done in this framework as well (see Golosov, Jones, and Terfilt (2007)). Finally, note that we assume that individuals, not couples, have children. This is done to simplify the development that follows.

⁷Formally, $f: \mathcal{P} \Rightarrow \{0, 1, \dots, \bar{f}\}$. We only consider feasible fertility plans—those for which $f(i^t) = 0 \Rightarrow f(i^t, i) = 0$ for all $i \in \mathcal{F}$. Then \bar{F} is the set of these feasible fertility plans.

technology is characterized by a production set $Y \subset \mathbb{R}^{k\infty}$ that describes all feasible input–output combinations. An element of the production set is denoted by $y \in Y$. We write $y = \{y_t\}_{t=0}^\infty$, where $y_t = (y_t^1, \dots, y_t^k)$ is the projection of the production plan onto time t .

An allocation is then given by a fertility plan, a consumption plan, and a production plan— (f, x, y) . We denote by A the set of all allocations and, for $i \in \mathcal{P}$, we use $A(i)$ to denote the set of all allocations in which i is born. When it is important to distinguish the choices individual i makes from those made by the other agents, we use the notation $(f(i), x(i); f(-i), x(-i))$.

We assume that each potential agent is described by both an endowment of goods and preferences. We use $e(i) \in \mathbb{R}^k$ to denote individual i 's endowment and note that $e(i)$ will be irrelevant in all that follows if $i \notin I(f)$. To simplify, we assume that preferences are described by a utility function, denoted by $u_i(f, x)$, which we allow to depend on the entire fertility and consumption plan components of the allocation. We do this to allow for the possibility of external effects across members of a family. For example, this specification allows utility to depend on fertility choices and the consumption of one's children etc. Below we will add an assumption that restricts utility to depend only on fertility and allocations within one's own dynasty.

We consider two possible assumptions for the domain of u_i :

ASSUMPTION 1: For each $i \in \mathcal{P}$, there is a well defined, real-valued utility function $u_i: A \rightarrow \mathbb{R}$.

ASSUMPTION 2: For each $i \in \mathcal{P}$, there is a well defined, real-valued utility function $u_i: A(i) \rightarrow \mathbb{R}$.

The difference between these two assumptions is that in the first, we assume that utility is well defined for all potential agents, even for plans in which they are not born. In the second, we assume that utility is only defined for an individual over those allocations in which he is born. We will use these different notions in our definitions of efficiency that follow.

There is a long-standing debate in the moral philosophy literature on what the utility of unborn people should be (see, for example, Singer (1993)). When considering preferences about adding new people to the status quo, there are three ways to think about this: (i) What are the preferences of the parents, siblings, and anyone else who feels potentially altruistic toward the newborn? (ii) How does the newly added person feel about this? (iii) What are the preferences of society as a whole? Parental preferences (i) are probably the least controversial concept and most models of endogenous fertility include some sort of altruistic preferences like this—either from parents to children, from children to parents, or both. This implies that there is a trade-off between having a child and not. Such preferences can also easily be derived from observed

choices.⁸ Other approaches to efficient fertility choice (like the social welfare approach of Blackorby, Bossert, and Donaldson (1995)) make explicit assumptions about societal preferences (i.e., (iii)), while we do not.⁹ Finally, do people have preferences about being born or not, and, if so, what are these preferences? These are hard questions. Although we sometimes assume that these preferences are well defined (i.e., Assumption 1 holds), we only use this assumption for the first of our efficiency concepts, \mathcal{P} -efficiency. For the second, which we call \mathcal{A} -efficiency, we only use Assumption 2. Thus, in \mathcal{A} -efficiency, the value of an additional child is based exclusively on the extra utility brought about to parents, grandparents, siblings, and so forth. In \mathcal{P} -efficiency, well defined preferences that include the state in which an individual is not born are required. However, the results that we prove (equilibrium fertility choice is \mathcal{P} -efficient) do not require assumptions on the form of these preferences—only that they exist.

Each individual that is born has a set of fertility and consumption plans that is feasible for him. For simplicity, we assume that this is the same for everyone and denote it by $Z \subset \{0, 1, \dots, \bar{f}\} \times \mathbb{R}^k$. The simplest version of this would have $Z = \{0, 1, \dots, \bar{f}\} \times \mathbb{R}_+^k$, so that any choice of fertility level and any nonnegative consumption is allowed. Because some models of fertility put restrictions on the joint choices of consumption and fertility (e.g., parents must care for their own children), we allow for the extra generality in Z . Most models of fertility also have a transferable cost of child production. Let $c(n) \in \mathbb{R}_+^k$ be the goods cost of having n children. We assume that this is the same for everyone for simplicity.

ASSUMPTION 3: *We have $c(0) = 0$ and $c(n)$ is strictly increasing in n .*

We can now define feasibility for this environment.

DEFINITION 1: An allocation (f, x, y) is *feasible* if:

- (i) $(f(i), x(i)) \in Z$, for all $i \in I(f)$;
- (ii) $\sum_{i \in I_t(f)} x(i) + \sum_{i \in I_t(f)} c(f(i)) = \sum_{i \in I_t(f)} e(i) + y_t$; for all t ;
- (iii) $y \in Y$.

3. EFFICIENT ALLOCATIONS

The formulation above turns models with an endogenous set of agents into models with a fixed set of potential agents, but with external effects in preferences, restrictions on what those potential agents that are not born can choose, and, possibly, domain restrictions on their utility functions. An advantage of this construction is that we can use, as a first cut, the normal notion of Pareto

⁸See Dasgupta (1994) for an ethical discussion of how parents *should* value fertility.

⁹See also Section 3.3 for an explicit comparison of our approach with theirs.

efficiency if utility functions are defined everywhere (i.e., if Assumption 1 is satisfied). We call this concept \mathcal{P} -efficiency, where \mathcal{P} refers to populations. This concept treats born and unborn people symmetrically and preserves the principle of “inclusiveness” of the usual Pareto criterion when comparing two allocations—every potential agent is “consulted” and one allocation dominates if and only if it is at least as good for all agents.

If utility functions are not defined for unborn agents over allocations in which they are not born (i.e., only Assumption 2 is satisfied), it is not possible to adopt such a strong notion of inclusiveness in the Pareto criterion. Indeed, if one goes to the opposite extreme and assumes that it is not possible to assign utilities to the unborn agents for any allocation in which they are not born, it is only possible, when comparing two allocations, to compare the utilities of agents that are alive in both. Our second notion of efficiency uses this reasoning exactly: when comparing two allocations, (f, x, y) and (f', x', y') , we compare the utilities of *all* agents that are alive in both, $I(f) \cap I(f')$. We call this second version \mathcal{A} -efficiency, because it focuses on *alive* agents. It is important to note that this does not mean that the consumption, and so forth of a potential child is not considered, rather that these enter only through the utility of other, alive, agents through familial external effects (e.g., parental altruism, and so forth).

As we will see later, many of our results hold for both definitions of efficiency, but we will also see that in specific applications the choice of concept matters.

3.1. Basic Concepts

\mathcal{P} -efficiency does not distinguish between agents who are born and not born in its treatment beyond what is implicit in feasibility and preferences. It is defined as follows.

DEFINITION 2: A feasible allocation (f, x, y) is \mathcal{P} -efficient if there is no other feasible allocation $(\hat{f}, \hat{x}, \hat{y})$ such that:

- (i) $u_i(\hat{f}, \hat{x}) \geq u_i(f, x)$ for all $i \in \mathcal{P}$;
- (ii) $u_i(\hat{f}, \hat{x}) > u_i(f, x)$ for at least one $i \in \mathcal{P}$.

Let \mathbb{P} denote the set of all \mathcal{P} -efficient allocations. If for any allocation (f, x, y) there exists some feasible allocation $(\hat{f}, \hat{x}, \hat{y})$ such that 2(i) and (ii) are satisfied, then we say that $(\hat{f}, \hat{x}, \hat{y})$ \mathcal{P} -dominates (f, x, y) . It follows that under Assumption 1, \mathcal{P} -domination is a well defined ordering of the feasible set. It is not complete (typically), but it is transitive and irreflexive. These are all properties of the usual notion of Pareto optimality in settings with fixed populations as well.

This definition seems to be the most natural extension of Pareto efficiency in the framework with endogenous fertility. It has, however, two important deficiencies. First, to choose which allocations are efficient, it is necessary that the preferences of the unborn agents be well defined—Assumption 1 must be satisfied. Unlike alive agents, whose preferences could be at least deduced from their observed choices, preferences of the unborn agents are inherently impossible to observe.¹⁰ Therefore, the set of efficient allocations will depend on an arbitrary choice of the preferences for the unborn. This leads to a second deficiency. One natural benchmark level of utility for the unborn is that being alive is always preferred. We can formalize this assumption in the following way:

ASSUMPTION 4: (a) For all $i \in \mathcal{P}$, there exists \bar{u}_i such that for all (f, x) , if $i \in \mathcal{P} \setminus I(f)$, then $u_i(f, x) = \bar{u}_i$.

(b) For all (f, x) and all i , if $i \in I(f)$, then $u_i(f, x) > \bar{u}_i$.

Note that if Assumption 4 is satisfied with $\bar{u}_i = 0$, then \mathcal{P} -efficiency satisfies the *Pareto-plus principle*¹¹: An allocation with an additional person enjoying a positive utility level is preferred to an allocation without the additional person but otherwise identical.

It is easy to see that under Assumption 4 it is impossible to have a population level that is too high. Any allocation with fewer agents will necessarily decrease the utility of the agents who were born under the original allocation but no longer are; therefore the new allocation cannot be \mathcal{P} -superior.

Because of this, Assumption 4 is quite strong. It is instructive to use this assumption in some examples and results below, but it will not be required for our main results.

Moreover, our second notion of efficiency overcomes these difficulties by treating born and unborn potential people asymmetrically. In this way, efficient allocations *do not* depend on preferences of the unborn or even on whether such preferences are defined at all; that is, only Assumption 2 needs to be satisfied (but it is also defined if Assumption 1 is satisfied).

DEFINITION 3: A feasible allocation (f, x, y) is \mathcal{A} -efficient if there is no other feasible allocation $(\hat{f}, \hat{x}, \hat{y})$ such that:

- (i) $u_i(\hat{f}, \hat{x}) \geq u_i(f, x) \quad \forall i \in I(f) \cap I(\hat{f})$;
- (ii) $u_i(\hat{f}, \hat{x}) > u_i(f, x)$ for some $i \in I(f) \cap I(\hat{f})$.

The definitions of the set of \mathcal{A} -efficient allocations \mathbb{A} and \mathcal{A} -domination are defined analogously to \mathcal{P} -efficiency.

¹⁰Note, however, that preferences of people that are not yet born can also not be deduced from observed choices. Yet it is a standard assumption made in overlapping generations models that utility functions for all (future) generations are well defined.

¹¹See Sikora (1978) and Dasgupta (1994).

This definition differs from \mathcal{P} -efficiency in that only a subset of the potential population is considered when making utility comparisons across allocations. An allocation is superior if no one who is alive in both allocations is worse off and at least one person alive under both allocations is strictly better off. Because utility comparisons are made only for the agents who are in fact born (i.e., $i \in I(f) \cap I(\hat{f})$), \mathcal{A} -efficiency has the added advantage of not requiring utility functions to be defined for agents who are not born. We call it \mathcal{A} -efficiency because only *alive* agents are considered. (Note that even agents that are not born count in \mathcal{A} -efficiency, at least indirectly, because they enter the utility functions of their parents, etc.) The disadvantage is that the set of agents considered in welfare comparisons depends on the two allocations being considered. This can, in some cases, cause cycles and, hence, nonexistence.¹² However, we show in Section 3.4 that generically (in utility functions) the set of \mathcal{A} -efficient allocations is nonempty.¹³

The notions of \mathcal{P} - and \mathcal{A} -efficiency extend the standard notion of Pareto efficiency. In particular, given any feasible allocation (f^*, x^*, y^*) , we can consider the standard Pareto ranking over allocations by holding the population fixed at $I(f^*)$. The next proposition shows that if (f^*, x^*, y^*) is \mathcal{P} -efficient (resp. \mathcal{A} -efficient), then (x^*, y^*) is a Pareto efficient allocation in the usual sense.

PROPOSITION 1: (a) *If Assumption 4(a) holds and if (f^*, x^*, y^*) is \mathcal{P} -efficient, then the consumption/production plan (x^*, y^*) is an allocation that is Pareto optimal among the agents in $I(f^*)$.*

(b) *If (f^*, x^*, y^*) is \mathcal{A} -efficient, the consumption/production plan (x^*, y^*) is an allocation that is Pareto optimal among the agents in $I(f^*)$.*

PROOF: Let (f^*, x^*, y^*) be a \mathcal{P} -efficient (\mathcal{A} -efficient) allocation. By way of contradiction suppose that there is some allocation (\tilde{x}, \tilde{y}) that is feasible given the set of alive people $I(f^*)$ that dominates (x^*, y^*) in the usual Pareto sense.¹⁴ It is immediate that, in this case, $(f^*, \tilde{x}, \tilde{y})$ necessarily \mathcal{A} -dominates (f^*, x^*, y^*) . That $(f^*, \tilde{x}, \tilde{y})$ also \mathcal{P} -dominates (f^*, x^*, y^*) follows from Assumption 4(a). Therefore, (f^*, x^*, y^*) could not be \mathcal{P} -efficient (\mathcal{A} -efficient). *Q.E.D.*

The converse of this proposition will not necessarily hold even if Assumption 4 holds. That is, even if an allocation is Pareto efficient in the usual sense

¹²Note that \mathcal{A} -domination need not be transitive.

¹³Conde-Ruiz, Gimenez, and Perez-Nievas (2004) propose a modification of \mathcal{A} -efficiency that requires symmetry among all people born in the same period. This modified concept guarantees existence, but is substantially less general because it does not allow for heterogeneity in preferences, endowments, or allocations at a point in time.

¹⁴Feasibility given a set of people is defined in the usual way.

holding the population fixed, it need not be either \mathcal{P} -efficient or \mathcal{A} -efficient, because welfare might be increased by changing the set of people alive.¹⁵

3.2. Examples

To illustrate our two notions of efficiency, we now consider two simple examples motivated by Becker and Barro (1988) and Barro and Becker (1989).

EXAMPLE 1: Consider a two-period example with only one parent, $\mathcal{P}_0 = \{1\}$. In period 0, there are e_0 units of a good that can be used either for consumption or for raising children. The cost of each child is $\theta > 0$. Parents care about own consumption and are altruistic toward each child as well. The utility function of the parent is

$$u_1(c(1), f(1); c(1, 1), \dots, c(1, f(1))) = \begin{cases} u(c(1)) + \beta \frac{1}{f(1)^\eta} \sum_{j=1}^{f(1)} u(c(1, j)), & \text{if } f(1) > 0, \\ u(c(1)), & \text{if } f(1) = 0, \end{cases}$$

where u is nonnegative, strictly increasing, and strictly concave, $0 < \beta < 1$ and $0 < \eta < 1$. The utility function of the i th potential child is given by

$$u_{(1,i)}(c(1), f(1); c(1, 1), \dots, c(1, f(1))) = \begin{cases} u(c(1, i)), & \text{if } 1 \leq i \leq f(1) \text{ (} i \text{ is born),} \\ \bar{u}, & \text{if } f(1) < i \text{ (} i \text{ is not born).} \end{cases}$$

In the example, we assume that Assumption 1 holds: utility of the child is well defined when not born. Note that without this assumption, \mathcal{P} -efficiency is not defined, but \mathcal{A} -efficiency is unchanged. Furthermore, we assume that each child, if born, has an endowment of the consumption good $e(1, i) = e_1 > 0$. To simplify, we assume that e_1 is not transferable.¹⁶ Then the possible utility levels for the parent are given by

$$W(f(1)) = u(e_0 - \theta f(1)) + \beta \frac{1}{f(1)^\eta} \sum_{1 \leq j \leq f(1)} u(e_1) = u(e_0 - \theta f(1)) + \beta f(1)^{1-\eta} u(e_1),$$

¹⁵Of course, if it is physically not feasible to change the set of people, then all three concepts coincide.

¹⁶We assume that each born period 1 child must consume her own endowment. Adding the possibility of redistributing the endowments of period 1 children increases the size of the sets of efficient outcomes in the usual way.

where $f(1) \in \{0, 1, \dots, \bar{f}\}$. We assume that $W(f(1))$ has a unique maximum f^* , with $0 < f^* < \bar{f}$. Furthermore let $e_0 > \theta \bar{f}$.

First consider the case where $u(e_1) > \bar{u}$. In this case, it is straightforward that no fertility level less than f^* is efficient (either \mathcal{A} or \mathcal{P}): increasing fertility to f^* from such a level *strictly* increases the utility of the parent and the added children, and does not lower the utility level of anyone. It also follows that any $f \in \{f^*, \dots, \bar{f}\}$ along with $c(1) = e_0 - \theta f$ gives a \mathcal{P} -efficient allocation. This is because any increase in fertility would necessarily lower the utility of the parent and any decrease would lower the utility of the children that are no longer born.

In contrast, f^* is the unique \mathcal{A} -efficient fertility level, because any fertility level higher than f^* is \mathcal{A} -dominated by f^* : moving to f^* strictly increases the utility of the parent and does not change the utility of the children that are still born.

If instead $u(e_1) < \bar{u}$, the set of \mathcal{P} -efficient allocations corresponds to all fertility levels in the set $\{0, \dots, f^*\}$, while the unique \mathcal{A} -efficient allocation still has $f = f^*$ as above.

In this example, the set of \mathcal{P} -efficient allocations is much larger than the set of \mathcal{A} -efficient allocations, a difference that holds more generally, as we will discuss below. The example shows that larger populations can be \mathcal{A} -dominated by smaller ones if reducing the size of the population does not lower the utility of those agents that are still born. Thus, \mathcal{A} -efficiency does not suffer from the difficulty pointed out above for \mathcal{P} -efficiency.

This example also illustrates another general property of the set of \mathcal{P} -efficient allocations: that although this set does vary with the assignment of utility levels to the unborn (i.e., the \bar{u}_i), there are typically some fertility plans that are \mathcal{P} -efficient no matter what the \bar{u}_i are. These are the fertility plans that maximize the utility of the time 0 parents.¹⁷

EXAMPLE 2: One might get the impression from Example 1 that \mathcal{A} -efficiency corresponds to maximizing the utility of the dynasty head. This is not true in general, however. Consider a slightly modified version of Example 1 in which goods from period 0 can be stored, with no loss, to period 1 and goods can be transferred among the period 1 agents. Feasibility here is captured in the two constraints

$$c(1) + f(1)\theta + \sum_{j=1}^{f(1)} c(1, j) \leq e_0 + f(1)e_1 \quad \text{and} \quad c(1) \leq e_0 - f(1)\theta.$$

¹⁷Technically, if a fertility plan is the *unique* maximizer of a weighted sum of utilities that puts positive weight only on agents in \mathcal{P}_0 , then that plan is \mathcal{P} -efficient for every assignment of the utilities of the unborn. Thus “typically” here means that it is necessary for the problem to have a unique solution, which is true generically in utility functions. Compare with Result 2 in the next section.

Again first consider the unique outcome that is best for the dynasty head: For simplicity, assume that $f^* = 1$ and $c^*(1) > 0$. Then $c^*(1, 1) \geq e_1$ follows from feasibility. This allocation is clearly \mathcal{A} -efficient. However, this is not the only \mathcal{A} -efficient allocation. Lowering consumption of the parent by δ and increasing the consumption of the child by the same amount will also lead to an \mathcal{A} -efficient allocation as long as $u(c^*(1) - \delta) + \beta u(c^*(1, 1) + \delta) > u(e_0)$.¹⁸ The logic is the same as with regular Pareto efficiency: there are two agents who disagree about the distribution of resources, and efficiency has nothing to say about redistribution; hence, many allocations are efficient.

So far, one could still suspect that *fertility* in any \mathcal{A} -efficient allocation is always equal to the most preferred choice of the dynasty head. However, this is not true either. If $f^* > 1$, then there are also typically \mathcal{A} -efficient allocations with $f < f^*$. To see this, let $e_0 = 100$, $e_1 = 0$, $\theta = 24$, $\beta = 1$, $\eta = 0$, and $u(c) = \sqrt{c}$. For these parameters, the parent's most preferred allocation is to have two children and split resources evenly, that is, $c(1) = c(1, 1) = c(1, 2) = \frac{100-48}{3}$, which gives utility 12.48 to the parent, and is \mathcal{A} -efficient. Now consider the allocation that maximizes the parent's utility *conditional* on having only one child: $\hat{c}(1) = \hat{c}(1, 1) = \frac{100-24}{2} = 38$. Clearly, this allocation is strictly preferred by the child and worse for the parent, whose utility under this allocation is only 12.33. To see that this allocation is also \mathcal{A} -efficient, note that it cannot be \mathcal{A} -dominated by the allocation with zero children, because this would give only utility $\sqrt{100} = 10$ to the parent. It also cannot be \mathcal{A} -dominated by any allocation with two children, because any such allocation would have to give at least 38 to the first child, which leaves only $\frac{100-38-48}{2} = 7$ each for the parent and the second child, and parental utility decreases to 11.46.

There are also other types of examples where \mathcal{A} -efficiency differs from dynastic head maximization. These include examples where children prefer to be in families with a large number of children (so that fertility levels higher than f^* are \mathcal{A} -efficient) and examples where parents and children do not have the same utility functions over the consumption of the child (i.e., there is a time consistency problem within the dynasty—altruism is imperfect). To save on space, we do not include any examples of this sort here.

3.3. Comparison to Critical Level Utilitarianism

An alternative approach to optimal population appears in the social choice literature (see Blackorby, Bossert, and Donaldson (1995, 2002, 2005)). This literature derives characterization theorems of the functional form of societal welfare functions (SWF) under a variety of alternative specifications of axioms. Blackorby, Bossert, and Donaldson (henceforth BBD) derive a SWF of the

¹⁸If δ is such that the condition is violated, then the parent strictly prefers zero children and hence the allocation is not \mathcal{A} -efficient.

“critical level utilitarianism” form. Under this form, the value to society of an allocation (f, x) is given by

$$W(f, x; \alpha) = \sum_{i \in I(f)} [u_i(f, x) - \alpha],$$

where $u_i(f, x)$ is i 's utility from being alive under the allocation (f, x) .¹⁹ BBD assume that the set of potential people is finite, which guarantees that this sum is well defined and that one alternative available is to have no people alive. However, these assumptions make it difficult to apply the concept to many positive theories of fertility, which usually allow for an infinite number of people and are conditional on an initial population being alive in all states (e.g., Becker and Barro (1988)).

The parameter α is interpreted as an ethical parameter. The idea is that a new person contributes to social welfare only if her utility is at least α . If a person lives a miserable life, that is, $u_i(f, x) < \alpha$, then the assumption is that it would be better for society if person i never existed. Note that when $\alpha = 0$, critical level utilitarianism reduces to the Benthamite welfare function.

One important finding is that with this specification, the “repugnant conclusion”—that the SWF ranks allocations with large numbers of people but low utility higher than allocations with few people and high levels of utility—does not hold if $\alpha > 0$.²⁰ This is also true of both \mathcal{P} - and \mathcal{A} -efficiency. That is, adding more children is not always a \mathcal{P} (resp. \mathcal{A}) improvement, because typically this will decrease the level of consumption of the parent and at some point the utility of the parent will fall because of this. As discussed above, large populations will be \mathcal{P} -efficient (but only when Assumption 4 holds), but they will not be the only \mathcal{P} -efficient allocations. Typically, large populations will not be \mathcal{A} -efficient, even when parents get positive utility benefits from having children. That is, if individuals have selfish motives for limiting family size (e.g., own consumption), allocations that exhibit restricted fertility will be efficient (both \mathcal{P} and \mathcal{A}). This is similar to the point made by Hammond (1988).

The goal of approaches based on SWF is to determine *the* optimal population size. \mathcal{A} - and \mathcal{P} -efficiency are different and complementary in that with them, we are trying to trace out the analogue of the Pareto frontier. \mathcal{P} -efficiency is a natural extension of Pareto efficiency and it relates to critical level utilitarianism just as standard Pareto efficiency relates to standard utilitarianism. It is important to note that Pareto optimality is inherently a very different concept from social welfare maximization. Typically the set of Pareto

¹⁹Critical level utilitarianism is a special case of “generalized critical level utilitarianism,” where the social welfare function is defined as $\sum_{i \in I(f)} [g(u_i(f, x)) - g(\alpha)]$ for some increasing and continuous function $g(\cdot)$ and some scalar α .

²⁰This literature normalizes utility to zero for the case of “neutrality” with the interpretation that life is worth living when utility is strictly positive.

optima is very large and is implicitly agnostic about alternative welfare distributions across agents. On the other hand, the SWF maximizer (with some assumptions) is unique and does make judgements about alternative distributional arrangements. This requires the assumption of interpersonal comparability of utility and hence, utility functions are necessarily cardinal—a strong assumption that is not required for notions of efficiency.²¹

A weakness of critical level utilitarianism is the necessity to define the ethical parameter α . This is a weakness that is, to some extent, shared by \mathcal{P} -efficiency. It is easy to show²² that if $\alpha = \bar{u}$, then any allocation that maximizes $W(f, x; \alpha)$ is also \mathcal{P} -efficient.²³ \mathcal{A} -efficiency gets around this weakness because it requires no additional specification of ethical parameters or preferences of people who are not alive. We therefore believe that \mathcal{A} -efficiency might actually be useful as a way to offer some guidance in choosing α . This can be best illustrated in an example.

Recall Example 1 as outlined in Section 3.2. In that example, assuming that $u_{(1,j)}(e_1) > \bar{u}$, the set of \mathcal{P} -efficient fertility levels is given by $\mathbb{P}_f = \{f^*, \dots, \bar{f}\}$, while $\mathbb{A}_f = \{f^*\}$. Given a critical level α , let $f(\alpha)$ be the fertility level that maximizes $W(f, x; \alpha)$ subject to feasibility. We find that in the example, *any* level of fertility can be rationalized as optimal according to critical level utilitarianism for some α . This is not true for \mathcal{P} -efficiency under the assumption that people prefer being alive over not being born. Note that the set of \mathcal{P} -efficient allocations does not change with \bar{u} as long as \bar{u} remains below $u(e_1)$. At $u_{(1,j)}(e_1) = \bar{u}$, \mathbb{P}_f becomes $\mathbb{P}_f = \{f^*\}$ and for $u_{(1,j)}(e_1) < \bar{u}$, \mathbb{P}_f is $\{0, 1, \dots, f^*\}$. Thus, both BBD optimality and \mathcal{P} -efficiency are sensitive to the choice of a parameter that would be difficult to identify. Note, however, that independent of any assumptions about the size of \bar{u} , the unconstrained choice of fertility by the parent, f^* , is always \mathcal{P} -efficient. It is also \mathcal{A} -efficient, but will not in general be BBD optimal. This is an example of the first welfare theorem that we will prove in the next section.

In the example, because there are no external effects that would suggest that privately chosen fertility is too high, it would be difficult to rationalize any fertility level below f^* as being reasonable. We thus find it hard to defend critical levels α above the one that delivers f^* as a reasonable ethical choice. In other words, if parents believe that the life of their children is worth living, why should society object to this? We therefore find \mathcal{A} -efficiency to be the more intuitive concept and believe that \mathcal{A} -efficiency can be useful in offering society some guidance as to what critical level α to choose; that is, choose α such that $f(\alpha) = f^*$.

²¹It is straightforward to check that both \mathcal{P} - and \mathcal{A} -efficiency are invariant to arbitrary, monotone transformations of utility functions of any subset of the agents.

²²This follows from comparing the welfare function (1) with Result 1 in Section 3.4.

²³The reverse is not generally true, because the set of \mathcal{P} -efficient allocations is typically large.

Recall that in this example the unique \mathcal{A} -efficient allocation can be found by solving

$$\max_f u(e_0 - \theta f) + \beta f^{1-\eta} u(e_1).$$

The BBD social welfare function for this example is

$$\max_f [u(e_0 - \theta f) - \alpha] + [\beta f^{1-\eta} u(e_1) + f[u(e_1) - \alpha]].$$

By comparing these two expressions, it is clear that the maximizers will be identical if and only if $\alpha = u(e_1)$. For any critical level, $\alpha > u(e_1)$, “society” has more stringent criteria than parents for whether a life is worth living and, hence, $f(\alpha) < f^*$, which seems hard to justify.

Adding technological progress to the example, it is easy to show that there is no constant α that makes the BBD-optimal fertility level \mathcal{A} -efficient. In other words, an equilibrium allocation in a world where consumption naturally grows over time would never be BBD optimal. We interpret this as a weakness of critical level utilitarianism because it implies that equilibrium fertility choices are always nonoptimal even when there are no externalities and no reasons to believe that privately chosen actions are inefficient. One way around this weakness would be to allow α to differ across generations. In particular, as argued above, one could use the \mathcal{A} -efficiency concept to find the appropriate sequence of α 's.

Given that BBD derived their SWF as the unique function that satisfies several (desirable) axioms, one might wonder which axioms are violated in our approach. The answer is continuity, completeness, anonymity, and, in the case of \mathcal{A} -efficiency, transitivity of the societal preferences. The orders defined by our concepts of \mathcal{A} - and \mathcal{P} -dominance violate these axioms partly in the same way that regular Pareto efficiency would also violate them. Furthermore the anonymity axiom is key to deriving the same critical level α for everyone. Anonymity might seem reasonable in a context where all people are potential and one wants to know who should ideally exist.²⁴ In our work, however, we make a clear distinction between the initial generation and potential future people. Moreover, the explicit dynastic setup and the possibility that parents care more about their own children than other people's children make anonymity less convincing in our setup.

3.4. Properties

In this subsection we briefly discuss the extent to which some standard properties of Pareto efficiency carry over into our context. We start with a partial

²⁴Dasgupta (1994) labels this the *genesis problem* and points out important differences with setups in which an initial set of people exists.

characterization of efficient allocations. We then discuss conditions that guarantee that the set of \mathcal{P} -efficient (resp. \mathcal{A} -efficient) allocations is not empty. Finally, we analyze the relationship between these two notions of efficiency. Because \mathcal{P} -efficiency is not defined unless Assumption 1 holds, it should be understood to hold in all the results that follow (similarly, we assume, without explicitly listing it, that *at least* Assumption 2 holds whenever \mathcal{A} -efficiency is being discussed).

We start with a partial characterization of \mathcal{P} -efficient allocations.

RESULT 1: *Pick any welfare weights $\{a(i)\}_{i \in \mathcal{P}}$ such that $a(i) > 0 \forall i \in \mathcal{P}$. Suppose (f^*, x^*, y^*) is a solution to the problem*

$$(1) \quad \max_{(f,x,y)} \sum_{i \in \mathcal{P}} a(i) u_i(f, x),$$

subject to feasibility and suppose that $\sum_{i \in \mathcal{P}} a(i) u_i(f^, x^*) < \infty$. Then (f^*, x^*, y^*) is \mathcal{P} -efficient.*

PROOF: By way of contradiction, assume that there exists a feasible allocation (f, x, y) that \mathcal{P} -dominates (f^*, x^*, y^*) , where (f^*, x^*, y^*) is a solution to (1). Then $u_i(f, x) > u_i(f^*, x^*)$ for at least one i and $u_i(f, x) \geq u_i(f^*, x^*)$ for all $i \in \mathcal{P}$. Summing up, we have $\sum_{i \in \mathcal{P}} a(i) u_i(f, x) > \sum_{i \in \mathcal{P}} a(i) u_i(f^*, x^*)$, a contradiction. *Q.E.D.*

In contrast to the usual characterization results, the weights $a(i)$ are required to be strictly positive for Result 1. The reason is that strict and weak Pareto efficiency do not necessarily coincide in this context because preferences are typically not strictly monotone in all goods.²⁵ In other words, in environments in which weak and strong Pareto efficiency coincide, Result 1 holds with weakly positive weights.

RESULT 2: *Pick any weights $\{a(i)\}_{i \in \mathcal{P}_0}$ such that $a(i) \geq 0 \forall i \in \mathcal{P}_0$. Suppose (f^*, x^*, y^*) is the unique solution to the problem*

$$(2) \quad \max_{(f,x,y)} \sum_{i \in \mathcal{P}_0} a(i) u_i(f, x),$$

subject to feasibility and suppose that $\sum_{i \in \mathcal{P}_0} a(i) u_i(f^, x^*) < \infty$. Then (f^*, x^*, y^*) is both \mathcal{A} -efficient and \mathcal{P} -efficient.*

PROOF: To prove \mathcal{A} -efficiency, let (f^*, x^*, y^*) be a solution to problem (2) and assume by way of contradiction that it is \mathcal{A} -dominated by (f, x, y) .

²⁵In particular, people typically do not receive utility from consumption in periods in which they are not alive.

Then there must exist a $j \in I(f^*) \cap I(f)$ such that $u_j(f, x) > u_j(f^*, x^*)$ and $u_i(f, x) \geq u_i(f^*, x^*)$ for all $i \in I(f) \cap I(f^*)$, that is, in particular for all $i \in \mathcal{P}_0$. Note that j cannot be in \mathcal{P}_0 because then (f^*, x^*) would not be a maximizer of (2). However, then we have $\sum_{i \in \mathcal{P}_0} a(i)u_i(f, x) = \sum_{i \in \mathcal{P}_0} a(i)u_i(f^*, x^*)$ but $(f, x, y) \neq (f^*, x^*, y^*)$, hence (f^*, x^*, y^*) is not unique, a contradiction. The proof for \mathcal{P} -efficiency is very similar. It follows from the observation that if a \mathcal{P} -dominating allocation exists, then there is a $j \in \mathcal{P} \setminus \mathcal{P}_0$ that is strictly better off, while the utility for everyone in \mathcal{P}_0 would be unchanged. However this immediately violates uniqueness. *Q.E.D.*

That uniqueness is required in Result 2 is unusual, but using this in conjunction with the fact that $\mathcal{P}_0 \subset I(f)$ for every feasible allocation gives the result, because any other plan must necessarily make some agent in \mathcal{P}_0 worse off. If the solution is not unique and there are two solutions with different sets of individuals born, individuals in future dates may not be indifferent between the two plans even though those in \mathcal{P}_0 are; hence, the argument given may not hold. It also follows from this result that the set of \mathcal{A} -efficient allocations is generically nonempty, namely, if the planner's problem given here does *not* have a unique solution, utility functions can be changed by a small amount so that a unique solution is guaranteed. Then, for these perturbed utility functions, the set of \mathcal{A} -efficient allocations is nonempty.

Note that Assumption 4 is not required for Result 2. In other words, an allocation that solves problem (2) is \mathcal{P} -efficient independently of the specification of the utility of the unborn. That is, although the entire set \mathbb{P} obviously depends on important value judgements about the utility of unborn people, there are typically some allocations that are *always* \mathcal{P} -efficient, no matter what the value judgement is.

From these two results and a few technical conditions to guarantee that solutions to the problems like those given actually have solutions, it follows that both \mathbb{P} and \mathbb{A} are nonempty.²⁶

RESULT 3: *Assume utility functions are continuous and uniformly bounded above and below, that $Z \subset \{0, 1, \dots, \bar{f}\} \times \mathbb{R}^k$ is closed, that $Y \subset \mathbb{R}^{k\infty}$ is closed in the product topology, and that the set of feasible consumption/production plans is bounded period by period.*

- (a) *Then the set of \mathcal{P} -efficient allocations \mathbb{P} is nonempty.*
- (b) *Generically, the set of \mathcal{A} -efficient allocations \mathbb{A} is nonempty.*²⁷

We turn now to the relationship between the set of \mathcal{A} - and \mathcal{P} -efficient allocations. Intuitively, one would expect that $\mathbb{A} \subseteq \mathbb{P}$ —as one need not (weakly)

²⁶See Golosov, Jones, and Tertilt (2007) for the formal proof.

²⁷“Generically” here means: if $\mathbb{A} = \emptyset$ for some choice of utility functions and endowments, then there is another choice of utility functions, uniformly within ε such that $\mathbb{A} \neq \emptyset$ with the same endowments.

improve the utility of *all* agents to “block” an allocation; hence, it is typically easier to find an \mathcal{A} -dominating allocation than a \mathcal{P} -dominating allocation. However, there is a counterbalancing effect. Sometimes it may be more difficult to \mathcal{A} -dominate an allocation because the set of people whose utility could potentially be strictly improved is smaller. Because of this, there might exist \mathcal{A} -efficient allocations that are not \mathcal{P} -efficient.

EXAMPLE 3: Consider a two-period, one good example with one parent and one potential child, each having an endowment of $e > 0$ units of the consumption good in the period they are alive. There is a technology that allows the transfer of goods between periods one for one. The cost of having a child is $\theta > 0$. The utility function of the parent is $u_1(c(1), f(1); c(1, 1)) = u(c(1)) + f(1)u(c(1, 1))$ and that of the potential child is $u_{(1,1)}(c(1), f(1); c(1, 1)) = f(1)u(c(1, 1))$. If the parameters are such that $2u(e - \theta/2) = u(e)$, then the parent is indifferent between having a child (with both consuming $c(1) = c(1, 1) = e - \theta/2$) and not having one, but the child’s utility is higher if born. Because of this, the allocation in which the child is born \mathcal{P} -dominates the one in which he is not, but it does not \mathcal{A} -dominate it. In this case, having the child is both \mathcal{P} - and \mathcal{A} -efficient, while not having the child is \mathcal{A} - but not \mathcal{P} -efficient.

Examples like this one arise due to a difference between Pareto efficiency and weak Pareto efficiency in this environment. This equivalence can break down in our context for several reasons: lack of strict monotonicity in all commodities, indivisible fertility choices, and external effects.²⁸ In cases where these two notions are the same it follows that $\mathbb{A} \subset \mathbb{P}$. Even if the two notions are not the same, it is “typically” true that “most” of \mathbb{A} is contained in \mathbb{P} .

To formalize this, we need some preliminary developments.

PROPOSITION 2: *If Assumption 4 holds, if $(f, x, y) \in \mathbb{A} \setminus \mathbb{P}$, and if the allocation $(\hat{f}, \hat{x}, \hat{y})$ \mathcal{P} -dominates (f, x, y) , then:*

- (i) $I(f) \subset I(\hat{f})$;
- (ii) $u_i(\hat{f}, \hat{x}) = u_i(f, x)$ for all $i \in I(f) \cap I(\hat{f})$;
- (iii) $u_i(\hat{f}, \hat{x}) > u_i(f, x)$ for some $i \in I(\hat{f}) \setminus I(f)$.

PROOF: Part (i) follows immediately from Assumption 4, which implies that any \mathcal{P} -dominating allocation always has weakly more people. Because $(f, x, y) \in \mathbb{A}$, it follows that $u_i(\hat{f}, \hat{x}) \leq u_i(f, x)$ for all $i \in I(f) \cap I(\hat{f})$, but because $(\hat{f}, \hat{x}, \hat{y})$ \mathcal{P} -dominates (f, x, y) , it must also be true that $u_i(\hat{f}, \hat{x}) \geq$

²⁸In particular, when Assumption 4 is satisfied, then preferences of the unborn are locally satiated and, hence, typically, weak and strong efficiency need not coincide. Thus, for these two to coincide, we would need, at a minimum, that utilities of the unborn depend on the consumption of their born relatives, even if only by a marginal amount.

$u_i(f, x)$ for all $i \in \mathcal{P}$. Together this implies part (ii). Then part (iii) follows from (ii) together with the fact that $(\hat{f}, \hat{x}, \hat{y})$ \mathcal{P} -dominates (f, x, y) . *Q.E.D.*

Proposition 2 shows that the set of alive people in every \mathcal{P} -dominating allocation is strictly larger and that those alive in both are strictly indifferent. If there was a way to increase the population *and* increase the utility of even one of the agents in the original allocation, the allocation in question could not be \mathcal{A} -efficient. We will use these facts heavily in the discussion that follows. Indeed, the requirement that all agents be exactly indifferent is what makes it “rare” for an allocation to be in $\mathbb{A} \setminus \mathbb{P}$, as we shall see.

For the remainder of this section, we assume that T is finite and that there is only one good. We also assume that goods are perfectly transferable across time (both forward and backward) and that this is the only form of production that is possible.²⁹ Given this, we can replace the production set and so forth with the following simple assumption on the aggregate feasibility constraint:

ASSUMPTION 5: *Assume that aggregate feasibility takes the form*

$$\sum_{i \in I(f)} (x(i) + c(f(i))) \leq \sum_{i \in I(f)} e(i).$$

Finally, we specialize the form of the utility functions:

ASSUMPTION 6: *Assume that the utility function of agent i in dynasty j is given by*

$$u_i(f, x) = \begin{cases} v_i(f_j) + \sum_{i' \in I(f_j)} u_{i'}(x(i')), & \text{if } i \in I(f), \\ \bar{u}_i, & \text{if } i \notin I(f), \end{cases}$$

where $u_{i'}$ is assumed to be strictly increasing, strictly concave, and C^1 , and v_i is strictly increasing in f_j .

Note that we are *not* assuming that $u_{i'} = u_{i''}$ and, hence, this formulation is quite general. Further note that, by construction, there are assumed to be no utility externalities across dynasties (this is an assumption we will make in more generality in the next section).

Now we are ready to state the main result with regard to the relationship between \mathcal{P} - and \mathcal{A} -efficient allocations:

²⁹The assumption that goods are freely transferable both forward and backward in time is a strong one. We conjecture that this is not necessary however, because, in general, at efficient allocations, price-taking agents always act as if goods are freely transferable across time at the rate of exchange given by the prices that support the allocation.

PROPOSITION 3: Assume that $(f^*, x^*) \in \mathbb{A} \setminus \mathbb{P}$ and that the following statements hold:

- (a) At least one \mathcal{P} -dominating allocation of (f^*, x^*) , (\hat{f}, \hat{x}) , does not strictly increase the population of every dynasty.
 - (b) For all $i \in I(f^*)$, $x^*(i) > 0$.
 - (c) Assumptions 4, 5, and 6 hold.
- Then, there exists a sequence $\{(f_n, x_n)\}$, $(f_n, x_n) \in \mathbb{P}$ such that $(f_n, x_n) \rightarrow (f^*, x^*)$.³⁰

See the [Appendix](#) for the proof.

The proposition shows that under relatively mild assumptions, \mathcal{A} -efficient allocations are either also \mathcal{P} -efficient or are arbitrarily close to allocations that are.

4. EFFICIENT EQUILIBRIUM FERTILITY: A FIRST WELFARE THEOREM

Our study of the properties of efficient allocations leads one naturally to wonder whether equilibrium allocations are \mathcal{P} - (or \mathcal{A} -)efficient. Because models of fertility choice involve external effects among individuals within the same dynasty (at least), equilibrium will naturally involve some element of strategic behavior. Because of this there is no presumption that equilibrium allocations will be efficient (e.g., Boldrin and Jones (2002)). There are some cases in which they are however. Indeed, this is true for the Barro and Becker model of fertility. Their original paper does not formally analyze the interaction between family members through game theoretic strategy sets and choice of equilibrium concept. In this section, we provide this formalization and go on to provide a theorem that shows that equilibrium outcomes are efficient in this model.³¹ Finally, we discuss how the key insight of the arguments can be generalized.

In the Barro–Becker model, it is assumed that each agent alive in period t , $i^t = (i^{t-1}, i_t)$, derives utility from his own consumption $x_t(i^t) \in \mathbb{R}$ and the utility of his children. Preferences of agent i^t are defined recursively by

$$U_t(i^t) = \frac{(x_t(i^t))^{1-\sigma}}{1-\sigma} + \beta(f_t(i^t))^\eta \int_0^{f_t(i^t)} U_{t+1}(i^{t+1}) di_{t+1}$$

³⁰We write $(f_n, x_n) \rightarrow (f^*, x^*)$ if $f_n = f^*$ for large enough n and $x_n \rightarrow x^*$ in the normal Euclidean sense.

³¹To make the characterization of equilibrium more tractable, Barro and Becker assume that fertility can take on any positive value, not just integers. The necessary modifications to our efficiency concepts to include the continuous fertility case are straightforward (see Golosov, Jones, and Tertilt (2007)).

for $\sigma \in (0, 1)$ and $\eta < 1$. Person i^t chooses his own consumption, $x_t(i^t)$, his fertility, $f_t(i^t) \in [0, \bar{f}]$, and a bequest vector for each of his children $b_t(i_{t+1}; i^t) \in \mathbb{R}$ subject to his budget constraint:

$$p_t(x_t(i^t)) + c_t f_t(i^t) + \int_0^{f_t(i^t)} b_t(i_{t+1}; i^t) di_{t+1} \leq p_t e_t(i^t) + b_{t-1}(i_t; i^{t-1}).$$

Here, c_t are childbirth costs per child. Note that the budget constraint includes the bequest that he received from his own parent, $b_{t-1}(i_t; i^{t-1})$. As before, we assume that the technology is characterized by a production set $Y \subset \mathbb{R}^\infty$ and that the equilibrium production plan maximizes profits.³²

The equilibrium concept is a mixture of a subgame perfect and competitive equilibrium. The strategy sets are as follows. Let h^{t-1} denote the history up to and including period $t - 1$. In period t player i^t must choose

$$s^{i^t} \in S^{i^t}(h^{t-1}) = \begin{cases} A_t(h^{t-1}), & \text{if } i_t \leq f_{t-1}(i^{t-1}), \\ \{(0, 0, 0)\}, & \text{if } i_t > f_{t-1}(i^{t-1}), \end{cases}$$

where $A_t(h^{t-1}) = \{(x_t(i^t), f_t(i^t), b_t(\cdot; i^t)) | p_t(x_t(i^t) + c_t(f_t(i^t))) + \int_0^{\bar{f}} b_t(i; i^t) di \leq p_t e_t(i^t) + b_{t-1}(i_t; i^{t-1})\}$. Thus, if i^t is not born, he has no choices to make.

Period t utility for player i^t is given by

$$U_{i^t} = \frac{(x_t(i^t))^{1-\sigma}}{1-\sigma} + \beta(f_t(i^t))^\eta \int_0^{f_t(i^t)} \left[\frac{(x_{t+1}(i^{t+1}))^{1-\sigma}}{1-\sigma} + \beta(f_{t+1}(i^{t+1}))^\eta \right. \\ \left. \times \int_0^{f_{t+1}(i^{t+1})} \left[\frac{(x_{t+2}(i^{t+2}))^{1-\sigma}}{1-\sigma} + \dots \right] di_{t+2} di_{t+1} \dots di_{t+1} \right]$$

In an equilibrium, the sequence of consumption, fertility, and bequest plans should be a subgame perfect equilibrium (SPE) of this infinite horizon game. Of course, there are typically many SPE's of infinite horizon games that involve different threats of punishments off the equilibrium path. There is no easy way to select among these different equilibria, but one common selection criterion is that it not be too dependent on the assumption that time lasts forever. That is, it should be the limit of the equilibria of the finite horizon truncations of the infinite horizon game. That is, we will look at a sequence of economies where the generation T players cannot have kids and look at the limiting allocations as $T \rightarrow \infty$.³³

³²Throughout this section, we assume that Y is a convex cone that contains 0 and, hence, we ignore profits.

³³Arguments similar to those in Fudenberg and Levine (1983) can be used to show that the limit of the SPE outcomes of the finite horizon truncations of this game are SPE outcomes of the infinite horizon game.

DEFINITION 4: An equilibrium is a sequence of prices p^* , allocation and bequest rules for each dynasty, $i \in \mathcal{P}_0$, $\{(x_t^*(i'), f_t^*(i'), b_t^*(\cdot; i'))\}_t$, and a production plan y^* such that the following statements hold:

- (i) For each i , given p^* , $\{(x_t^*(i'), f_t^*(i'), b_t^*(\cdot; i'))\}_t$ is the limit of the subgame perfect equilibrium outcome of the finite horizon dynasty game.
- (ii) Given p^* , y^* maximizes profits, that is, $p^*y^* \geq p^*\hat{y} \forall \hat{y} \in Y$.
- (iii) The allocation is feasible.

THEOREM 1: *The equilibrium allocation from the Barro and Becker model is both \mathcal{P} - and \mathcal{A} -efficient.*

The proof of this result proceeds in three steps. The first step is to show that, for every finite horizon truncation, there is a unique SPE outcome and to characterize it. This equilibrium allocation has an important property, which is that it solves a maximization problem in which the time 0 parent chooses everything for his dynasty subject to a single, dynastic budget constraint. Thus, the equilibrium bequest strategies are such that, on the equilibrium path, the time 0 player can act as if he can move wealth freely between any and all of his descendants at market determined prices. It follows that the limiting allocation has a similar property. The second step is to show that the solution to this dynastic maximization problem is unique, which immediately implies that any other allocation in the budget set will make the time 0 parent strictly worse off. The third step is to argue that if an allocation is efficient (either \mathcal{P} or \mathcal{A}) within a dynasty and there are no external effects across dynasties then, overall, the allocation is \mathcal{P} - and \mathcal{A} -efficient. See Golosov, Jones, and Tertilt (2004) for a more detailed proof of this result.

The key insights from the Barro–Becker model can be generalized considerably. Indeed, any allocation for which each dynasty behaves efficiently internally and treats prices parametrically will be \mathcal{P} - and \mathcal{A} -efficient.

THEOREM 2: *Assume that preferences are monotone increasing in consumption and fertility within a dynasty, and that there are no cross-dynasty external effects. Consider a price vector p^* and an allocation (x^*, f^*, y^*) such that:*

- (i) *for each dynasty i , $i \in \mathcal{P}_0$, (x_i^*, f_i^*) is \mathcal{P} - (resp. \mathcal{A} -)efficient for dynasty i given its dynastic budget constraint*

$$\sum_t p_t^* \sum_{j \in \mathcal{P}_t \cap I(f_j)} (x(j) + c(f(j))) \leq \sum_t p_t^* \sum_{j \in \mathcal{P}_t \cap I(f_j)} e(j);$$

- (ii) *y^* maximizes profits at prices p^* , i.e., for all $y \in Y$, $p^*y \leq p^*y^*$;*
 - (iii) *(x^*, y^*, f^*) is feasible.*
- Then (x^*, y^*, f^*) is \mathcal{P} - (resp. \mathcal{A} -)efficient.*

The proof closely follows the standard proof of the first welfare theorem and is omitted. Note that the theorem *assumes* that dynasties are maximizing

subject to an infinite horizon budget constraint. This assumption also assures that the Samuelson inefficiency is not a problem here.

This result allows one to divide the efficiency question into two pieces: efficient transfer systems within a family and efficient trade across dynasties. This is exactly the outline followed in the proof of Theorem 1 above. A second example along this line is at the opposite extreme from the Barro and Becker world where preferences are aligned between parent and child. Suppose that there are no restrictions on preferences across generations, but that a rich set of bequest contracts is available. In the extreme, if bequests can be made conditional on all actions, then all time 0 parents can provide the correct incentives to all of their descendants to attain the maximum value for their own utility. It follows from the theorem that fertility choices will be efficient in this setting too.

5. APPLICATIONS

In many discussions, it is taken as a given by policy makers that fertility is too high in developing countries and too low in some developed countries.³⁴ Some governments provide free family planning and abortion services to discourage fertility, while others give large subsidies to encourage fertility. Few reasons are typically given for this view, although several auxiliary concerns are mentioned. These include the overall scarcity of factors as well as the role of population size and density in determining pollution.³⁵ In this section, we use the tools developed above to identify which of these concerns do and do not give rise to inefficient population growth rates. We find that scarce factors do not cause fertility to be inefficient, whereas global external effects do lead to inefficiencies. As pointed out in Section 3.1, there are typically never too many people in the \mathcal{P} -sense, and this will show up in some of the examples presented below.

5.1. *Land Scarcity*

In the policy debate it is often argued that because resources are scarce, fertility decisions affect society as a whole and should, therefore, not be left entirely to individuals. The logic provided is that parents do not take into account that an extra child decreases the amount of these scarce resources per capita. This leads to a discrepancy between private and social costs of children. Hence, an inefficiency might arise.³⁶

³⁴See, for example, Financial Times (2004).

³⁵Hardin (1968) argued that the “tragedy of the commons” leads to overpopulation. See also Becker and Murphy (1988) for a discussion of situations in which equilibria may be inefficient.

³⁶Many of those involved in the population debate are not economists. Because of this, they do not carefully distinguish between true and pecuniary externalities. As a by-product, they of-

In this section we argue that this logic is incorrect. The effect of reducing per capita income by adding an additional child (by increasing the aggregate labor supply) is analogous to the effect that an individual's increase in labor supply has on aggregate labor and thereby wages. These effects are channeled through prices and therefore do not lead to an inefficiency. Thus, this is an example of a *pecuniary externality*.

To see this, consider an example in which there are three goods in each period. The first is land, the second is time, and the third is a consumption good. All agents are endowed with one unit of time, which they supply inelastically to firms if they are born. Those agents alive in period 0, indexed by $i = 1, \dots, N$, are also endowed with holdings of land, A^i . Let $\bar{A} = \sum_{i \in \mathcal{P}_0} A_i$. These holdings are sold to the firm and subsequently used forever. The production function is static: $y_t = F(A, \ell^f)$, where F is assumed to be constant returns to scale in land and labor input.

Profit maximization on the part of the firm then implies that the equilibrium price of land traded in period 0 is given by $q_0 = \sum_t F_A(\bar{A}, N_t) p_t$, where N_t is the size of the population in period t and p_t is the period 0 price of one unit of the consumption good in period t . Similarly, the real wage rate must be $w_t/p_t = F_\ell(\bar{A}, N_t)$.

Thus, in keeping with intuition, if, for whatever reason, $\hat{N}_t > N_t$ for all t and p_t is held fixed, the sale price of land (and the implicit rental price as well) is higher while the equilibrium real wage rate must be lower. That is, because land is scarce, if parents choose to have more children, real wages must be lower. In this sense, one parent would, across equilibria, lower the realized wage for all children by increasing his fertility choice. In this sense, there is crowding of scarce resources.

Despite this fact, it does not follow that equilibrium outcomes in models with scarce factors and endogenous population are necessarily inefficient. In fact, Theorems 1 and 2 include scarce factors and hence, cover examples like this.

5.2. Problems Across Dynasties (Pollution)

Our theory also points to situations when equilibria are inefficient. The results from Section 4 rely on the assumption that there are no external effects

ten go back and forth between arguing that population is too high simply because of crowding existing resources and because of taxing the ability of the environment to absorb pollutants. For an example, see the interview with Paul Ehrlich on Uncommon Knowledge (available at <http://www.hoover.org/publications/uk/2933321.html>) where he states "...you're overpopulated when you no longer can live on your interest, when you've got to live on your capital. And the three main forms of capital that we're getting rid of very, very rapidly at today's density and today's consumption patterns are deep rich agricultural soils, biodiversity, which is critical, and maybe the most short-term critical is our supplies of groundwater everywhere, which are being overdrafted." See also Ehrlich and Ehrlich (2002) and Dasgupta (2003) on crowding and population externalities.

across dynasties. Many policy debates implicitly or explicitly question the validity of this assumption. We now discuss some of these arguments.

One of the most frequently discussed reasons for a negative effect of a high population level is related to pollution and other adverse effects each agent may have on others. It is not clear, though, that such arguments justify policies that discourage fertility. For example, one might expect that standard Pigouvian taxes alone could restore optimality. In the Technical Appendix,³⁷ we examine this issue in the context of a simplified two-period version of the Barro–Becker model where external effects arise from pollution as a by-product of period 2 production.

We show that the equilibrium allocation without taxation is inefficient in two ways. First, there is too much output in period 2 (in both the \mathcal{P} - and \mathcal{A} -senses). This is the standard external effect. A standard Pigouvian tax on production leads to a Pareto improvement. It also achieves efficient allocations in the \mathcal{P} -sense. Even with this Pigouvian tax, however, the new allocations are not \mathcal{A} -efficient. The second inefficiency arises because the fertility is “too high.” Each parent, by having children, adversely affects other parents through the pollution thereby created. This external effect is not internalized by the Pigouvian tax in the second period. Thus, endogenous fertility adds an additional dimension to the standard pollution problem: Parents exacerbate the pollution problem by having too many children and a child tax, in addition to the Pigouvian pollution tax, will, in general, be needed.³⁸ However, such a tax will not typically lead to a \mathcal{P} -dominating allocation since it decreases the utility of children who are not being born because of the tax. This example shows that whether fertility is efficient when only a pollution tax is used depends on the particular notion of efficiency one uses.

This reasoning needs to be adjusted if the direction of the external effects is reversed (for example, if they arise due to knowledge spillovers). A higher number of children is beneficial for both new and existing people, so that the equilibrium allocation without child subsidies is not only \mathcal{A} - but also \mathcal{P} -inefficient.

Yet another plausible externality could arise when there is heterogeneity in the degree of altruism toward one’s children and some people derive disutility from seeing other parents neglect their children. It is easy to see that equilibrium fertility in such a case could be \mathcal{A} -inefficiently high and that an \mathcal{A} -superior allocation would involve some people compensating others for not having children. Alternatively, such an externality could provide an efficiency rationale for existing policies, such as mandatory schooling and parental leave policies.

Other examples of the failure of the first welfare theorem in this environment arise when key markets are missing. One can imagine many examples that are relevant in fertility settings (for example, the lack of insurance against

³⁷Golosov, Jones, and Tertilt (2007).

³⁸This conclusion (and the example we analyze) is similar to that found in Harford (1998).

the risk of not being able to have children). A particularly interesting example involves private information about expected lifetimes. This is a common explanation given for the relative sparsity of annuity markets. This may lead parents to have too many children, because parents use children as an alternative to annuity contracts. In other words, an \mathcal{A} -superior allocation would involve fewer people with better insurance across dynasties. The missing markets problem is similar to the pollution externality discussed above. In both cases, dynasties may well be maximizing and yet equilibrium fertility is too high due to a problem in the economy as a whole.

5.3. Problems Within a Dynasty (Drugs)

We now give an example of a game among dynasty members that leads to an equilibrium outcome that is not optimal for a dynasty. In this example, inefficiency arises from the lack of commitment between two family members, one of which is not born yet.

There is one initial old person and one potential child, $\mathcal{P} = \{1, (1, 1)\}$. The parent derives utility from her own consumption and from the consumption of her child: $u_1 = u(c_1) + f_1 \beta u(c_{(1,1)})$, where $u(\cdot)$ is strictly concave. The child has preferences over consumption, $c_{(1,1)}$, and drugs, $d_{(1,1)}$, $u_{(1,1)} = c_{(1,1)} + \gamma d_{(1,1)}$. People in each period are endowed with one unit of time. A static technology converts labor into consumption and drugs, $c + d \leq F(\ell) = w\ell$. It costs θ units of the consumption good to produce a child. Suppose $\gamma > 1$. Then the optimal strategy for $(1, 1)$ is to consume only drugs, if born. Then the following is a subgame perfect equilibrium allocation: $z = \{c_1 = w, f_1 = 0, c_{(1,1)} = 0, d_{(1,1)} = 0\}$. The reason for zero equilibrium fertility is that knowing that his child will be a drug addict, the parent prefers not to have a child. However, note that, assuming θ is not too large, z is not \mathcal{P} -efficient, because the allocation $Z = \{c_1 = w - \theta, f_1 = 1, c_{(1,1)} = w, d_{(1,1)} = 0\}$ is \mathcal{P} -superior.³⁹

Note that the above inefficiency does not disappear with negative bequests. Instead, a tax-and-transfer system is required so that the parent can discourage the use, by the child, of the good the parent does not want the child to consume. More subtle disagreements between generations can cause similar problems. A very natural form of dissent would arise if parents and grandparents differ in their evaluation of their child/grandchild.⁴⁰

Note, however, that time inconsistent preferences between parents and children do not have to lead to an inefficiency. It is easy to construct an example where parents and children disagree, but the equilibrium is still efficient, because any other allocation would make the child worse off. This point is related to an argument made in Section 3.2, where it was shown that efficiency need not coincide with utility maximization of the parent. Disagreement between

³⁹The alternative allocation is also \mathcal{A} -superior.

⁴⁰An example of this type, but with exogenous fertility, was given by Phelps and Pollak (1968).

parents and children may simply lead to an equilibrium allocation that favors the child (because the child chooses second), but this need not be inefficient.

6. CONCLUSION

In this paper, we have presented two extensions of the notion of Pareto optimality for models in which fertility is endogenous: \mathcal{P} -efficiency and \mathcal{A} -efficiency. We have shown that the most popular economic model of fertility choice (Barro and Becker (1989)) gives rise to efficient allocations—population size is efficient. More generally, if external effects are confined to the dynasty and dynasties make optimal decisions, fertility choices will be efficient. Finally, we have shown that the presence of external effects can cause individually optimal fertility choices to be suboptimal from a social point of view and that this bias depends on the direction of the external effect.

Our analysis suggests there are two types of reasons for inefficiencies when fertility is endogenous. First, the assumptions of the first welfare theorem might not be satisfied for standard reasons such as global external effects, public goods, congestion effects, missing markets, and private information. Second, dynasties might not behave in a way that is optimal for the dynasty because of limitations on bequests or a lack of perfect altruism.

There are several issues that have not been addressed in the current paper, but seem interesting for future research. One is to extend the concepts to allow for uncertainty and then analyze interactions between fertility and missing markets (such as annuity markets) in a more serious way. Second, this paper assumes unisexual reproduction, whereas one would like to be able to address questions of marriage.

We also think that an analysis of existing fertility policies would be very interesting. Using our concepts might shed some light on the important policy debates on population that are now being waged. For example, some people argue that fertility is too low in many European countries. The arguments typically given are along the line that the social benefit of having children exceeds the private benefit, because, without children, labor supply will be too small in the future. This argument does not point to any particular reason for the theorems we have presented to not hold—no global external effects or particular difficulties for families to be making efficient decisions are mentioned—and, thus, it is reminiscent of the scarce factor example discussed above.⁴¹ Even with problems like these, the appropriate intervention depends on the exact nature of the imperfection. Thus, although it is possible that the conclusion is correct—perhaps because of the difficulty in leaving negative bequests—we believe that it is critical to identify the precise source of the inefficiency before a serious policy debate can be held.

⁴¹Of course, another rationale for intervention is that it allows governments to choose an efficient allocation different from the one that arises in equilibrium.

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Manuscript received March, 2005; final revision received December, 2006.

APPENDIX: PROOF OF PROPOSITION 3

For any given population I , let $C(I)$ be the total cost of child rearing with that population. Let $Y(I) \equiv \sum_{i \in I} e(i) - C(I)$. Let $Y^j(I)$ be the total resources available for consumption if we consider only the endowments of a dynasty j in the population I , that is, $Y^j(I) = \sum_{i \in D_j \cap I} e(i) - C^j(D_j \cap I)$, where $C^j(D_j \cap I)$ is the total cost of rearing the children born to dynasty j if I is the population. Because child rearing costs are additive, $Y(I) = \sum_j Y^j(I)$.

Consider any \mathcal{A} -efficient allocation (f^*, x^*) . Let (f_j^*, x_j^*) be the allocations in (f^*, x^*) that agents in dynasty j receive. By Assumption 6, there are no external effects across dynasties and, hence, for some wealth redistribution $T^* = (T_j^*)_{j \in \mathcal{P}_0}$ with $\sum_{j \in \mathcal{P}_0} T_j^* = 0$, the (f_j^*, x_j^*) , $j \in \mathcal{P}_0$, each solve the dynastic maximization problem

$$\begin{aligned}
 &V^j(I(f^*), T^*, u^*) = \max_{(f, x)} u_j(f, x) \\
 &\text{s.t. } u_i(f, x) \geq u_i^* \quad \text{for all } i \in I(f^*) \cap D_j \setminus \{j\}, \\
 &\quad \sum_{i \in I(f^*) \cap D_j} x_i \leq Y^j(I(f^*)) + T_j^*
 \end{aligned}$$

for $u_i^* = u_i(f_j^*, x_j^*)$. We also let $u_i^* = \bar{u}_i$ for all $i \in \mathcal{P} \setminus I(f^*)$. We denote the vector of utilities that arises in this way by $V(I(f^*), T^*, u^*) = (V^j(I(f^*), T^*, u^*))_{j \in \mathcal{P}_0}$.

Denote by $\alpha_j^* = (\alpha_i^*)_{i \in I(f^*) \cap D_j \setminus \{j\}}$ the vector of multipliers on the utility constraints on the problem (i.e., α_i^* is the multiplier on the constraint $u_i(f, x) \geq u_i^*$), and note that this problem can be rewritten as maximizing a weighted sum of utilities of those dynasty members in $I(f^*) \cap D_j$ with weights given by α_i^* for the members in $I(f^*) \cap D_j \setminus \{j\}$ and 1 for j himself.

LEMMA 1: *Consider any (f^*, x^*) that is in $\mathbb{A} \setminus \mathbb{P}$. Then, for any $j \in \mathcal{P}_0$, there exists another population I , $I(f^*) \subset I$, and an allocation (\tilde{f}, \tilde{x}) that solves*

$$\max u_j(\tilde{f}, \tilde{x})$$

$$\begin{aligned}
 \text{s.t. } & u_i(f, x) \geq \hat{u}_i \quad \text{for all } i \in I \cap D_j \setminus \{j\}, \\
 & \sum_{i \in I(f^*) \cap D_j} x(i) \leq Y^j(I(f^*)) + T_j^*.
 \end{aligned}$$

Moreover, the solution to this problem is such that $V^j(I(f^*), T^*, u^*) = V^j(I, T^*, u^*)$ for all j .

The proof follows from Proposition 2 and the discussion above.

Pick any $(f^*, x^*) \in \mathbb{A} \setminus \mathbb{P}$ and corresponding α^*, T^* . Let $I^* = I(f^*)$ be the population in that allocation. Let (\hat{f}, \hat{x}) be any allocation that \mathcal{P} -dominates (f^*, x^*) such that condition (a) of Proposition 3 is satisfied. We know that $I(f^*) \subset I(\hat{f})$. From Lemma 1, $V^j(I^*, T^*, u^*) = V^j(I(\hat{f}), T^*, u^*)$ for all $j \in \mathcal{P}_0$.

Note that any allocation (\hat{f}, \hat{x}) with a population larger than I^* must have less total resources available for consumption, $Y(I(\hat{f})) < Y(I(f^*))$. Otherwise, all agents in $I(f^*)$ could receive exactly the same consumption as under (f^*, x^*) , and this new allocation would clearly \mathcal{A} -dominate (f^*, x^*) . This implies that there must be some agent with a positive α^* weight such that $x^*(i) > \hat{x}(i)$. Using our assumption about utility functions (Assumption 6) this implies that the consumption of all agents with positive α_i^* in the dynasty also falls. To see this, consider any agent with positive α_i^* weight. By Proposition 2, utilities of all other agents in the dynasty either remain constant or increase. For the agent’s utility level to remain unchanged it must therefore be true that his consumption decreased.

Using the form of the utility function and the assumption that $x^*(i) > 0$ for all i , we can apply the envelope theorem,

$$\begin{aligned}
 (3) \quad V_{T_j}^j(I^*, T^*, u^*) &= \sum_{i' \in D_j \cap I^*} \alpha_{i'}^* \frac{\partial u_{i'}(x^*(i))}{\partial x(i)} \\
 &< \sum_{i' \in D_j \cap I^*} \alpha_{i'}^* \frac{\partial u_{i'}(\hat{x}(i))}{\partial x(i)} = V_{T_j}^j(I(\hat{f}), T^*, u^*),
 \end{aligned}$$

because $u_{i'}$ is strictly concave and $x^*(i) > \hat{x}(i)$.

Now we are ready to prove the main result:

PROOF OF PROPOSITION 3: Assume that dynasty j^* has new people under the allocation (\hat{f}, \hat{x}) , that is, $I(\hat{f}) \setminus I(f^*) \cap D_{j^*} \neq \emptyset$. Assume that (f_j^*, x_j^*) is supported by the transfers T_j^* and that the allocation maximizes the social welfare function with weights α^* . Take a sequence of T_n that converges to T^* with the restriction that $T_{j^*n} < T_{j^*}^*$ for all dynasties with more people under (\hat{f}, \hat{x}) and $\sum_j T_{jn} = 0$. Consider all the possible population sizes I with $I^* \subset I$. Because (f^*, x^*) is \mathcal{A} -efficient, it must be true that $V(I^*, T^*, u^*) \geq V(I, T^*, u^*)$

(here V^j is assumed to take the value $-\infty$ if the constraint set is empty in the maximization problem). Because $(f^*, x^*) \in \mathbb{A} \setminus \mathbb{P}$, this inequality must hold with equality for some I . Note that V is continuous in T at (I^*, T^*, u^*) as long as $\sum_{i \in D_j} x^*(i) > 0$ for all $j \in \mathcal{P}_0$, which is true by assumption. This implies that if $V^j(I^*, T^*, u^*) > V^j(I, T^*, u^*)$ for some j, I , then $V^j(I^*, T_n, u^*) > V^j(I, T_n, u^*)$ for all T_n close enough to T^* . Therefore, the allocations that solve $V^j(I, T_n, u^*)$ are \mathcal{A} dominated by those that solve $V^j(I^*, T_n, u^*)$. Consider any I such that $V^{j^*}(I^*, T, u^*) = V^{j^*}(I, T, u^*)$. By construction, $T_{j^*n} < T_{j^*}^*$. Thus, in a neighborhood of T^* , using (3), we have $V^{j^*}(I^*, T_n, u^*) > V^{j^*}(I, T_n, u^*)$ for all such j^* . Similarly, for all j such that $V^j(I^*, T^*, u^*) > V^j(I, T^*, u^*)$ it is still true that $V^j(I^*, T_n, u^*) > V^j(I, T_n, u^*)$. It follows that $(f_j(T_n), x_j(T_n))_{j \in \mathcal{P}_0}$ is a sequence of \mathcal{P} -efficient allocations that have a population size I^* . Because $(f_j(T_n), x_j(T_n))_{j \in \mathcal{P}_0} \rightarrow (f_j(T^*), x_j(T^*))_{j \in \mathcal{P}_0}$, this completes the proof. *Q.E.D.*

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