Motivation

1. Substantial increase in consumer bankruptcy filings.
   - 1.4 filings per 1,000 adults in 1970
   - 8.5 filings per 1,000 adults in 2002
   - Similar increases in Canada:
     from 0.2 per 1,000 adults in 1970 to 4.5 in 2004.

2. Debate about what caused the increase in filings.

3. Policy debate about reforming bankruptcy law.
Our Contribution

- Framework to evaluate proposed explanations for rise in consumer bankruptcy filings
  - Quantitative model of consumer bankruptcy
  - Numerical experiments in parameterized model

- Compare model implications of each story to key facts:

<table>
<thead>
<tr>
<th>Fact</th>
<th>1980-84</th>
<th>1995-99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 7 filings (% of HHs)</td>
<td>0.25%</td>
<td>0.83%</td>
</tr>
<tr>
<td>Unsecured Debt/Disposable Income</td>
<td>5%</td>
<td>9%</td>
</tr>
<tr>
<td>Average borrowing interest rate</td>
<td>11.5-12.7%</td>
<td>11.7-13.1%</td>
</tr>
<tr>
<td>Charge-off rate</td>
<td>1.9%</td>
<td>4.8%</td>
</tr>
</tbody>
</table>
Measurement of Unsecured Debt

- Unsecured debt/disposable income: 9% in aggregate data
- Negative net worth: 0.7-1.5% (SCF).

- Unsecured debt is better measure because:
  - many assets (partially) exempt from bankruptcy.
  - costly to seize assets (e.g. Standard & Poor’s estimate of foreclosure costs \(\sim 20-30\%\) of loan value).
  - Zinman (forthcoming): credit card debt underreported in SCF by \(\sim 50\%\).

- Further: credit card debt 9% of income and charge-off rate on credit cards 5%. If only negative net worth could be defaulted upon, would imply an implausibly high charge-off rate of 22.5% and an interest rate of more than 30%.
Proposed Explanations

1. Increase in earnings volatility
   (Barron, Elliehausen and Staten 2000)

2. Increase in expense risk (Warren and Warren Tyagi 2003)

3. Demographic changes in the population
   (Sullivan, Warren and Westbrook 2000)
   - Age composition (baby-boomers)
   - Marital status


5. Removal of interest rate ceilings (Marquette) (Ellis 1998)

6. Credit Market Innovation (Barron and Staten 2003)
Consumer Bankruptcy Law in the U.S.

- We focus on Chapter 7 (about 70% of all filings).
- Discharge unsecured debt in exchange for assets.
- Non-dischargeable: student loans, child support, alimony, etc.
- 6 years between filings
- Roughly 4 months process
- Court fees: $209, legal fees: $750-$1,500
Literature


- Closest to ours: Moss and Johnson (1999), Athreya (2004)

Summary of Our Results

- None of the explanations “works” individually.
- Can match all three key facts with a combination of:
  - Decline in stigma
  - Decline in transaction cost of lending
- Uncertainty based stories play small role quantitatively.
- Demographic changes: not important quantitatively.
- *Marquette*: not a main driving force.
A Model to Evaluate Stories

- Stochastic life cycle model
- Two types of idiosyncratic uncertainty:
  - Income shocks
  - Expense shocks
- Incomplete markets:
  Non-contingent debt only
  Consumers can declare bankruptcy.
- Equilibrium interest rate incorporates default risk,
  → interest rate depend on age, current income, total debt.
The Model: Households

- $J$-period lived households
- Preferences represented by:
  \[
  \sum_{j=1}^{J} \beta^{j-1} u(c_j)
  \]

- Expense Shocks
  - Exogenous increase in household’s debt
  - Idiosyncratic expense shock: $\kappa \in K$, iid
  - $K$ finite set of possible expense shocks
- Stochastic Labor Income: $y_j^i = z_j^i \eta_j^i \bar{e}_j$
Bankruptcy Punishments

1. Cannot save or borrow in default period.
   - Captures seizure of assets.

2. Cannot file following period.
   - Captures 6 year waiting period.


4. Fraction $\gamma$ of earnings is garnisheed.
   - Lenders receive $\Gamma = \gamma y$. 

Rise in Bankruptcies – p. 13/41
The Model: Financial Markets

- Incomplete markets: one-period non-contingent bonds only.
- Interest rate on savings exogenous: $r^s$.
- Risk-free borrowing: $q^b = \frac{1}{1+r^s+\tau}$, where $\tau$ is (proportional) transaction cost of making loans.
- Perfectly competitive financial markets.
  - Full information: Default probability $\theta(d, z, j)$ is common knowledge.
  - Zero expected profits on each loan.
  - Risk adjusted bond price:
    \[ q(d, z, j) = (1-\theta(d, z, j))q^b + \theta(d, z, j)E\left( \frac{\Gamma(z', j + 1)}{d + \kappa'} \middle| I = 1 \right) q^b \]
- Usury law: If $q(d, z, j) < \frac{1}{1+\tau}$, then $q(d, z, j)$ is set to 0.
Consumer Problem

\[ V_j(d, z, \eta, \kappa) = \max_{c,d'} \left[ u(c) + \beta E \max \{ V_{j+1}(d', z', \eta', \kappa'), \bar{V}_{j+1}(z', \eta') \} \right] \]

s.t. \( c + d + \kappa \leq \bar{e}_j z \eta + q^b(d', z, j)d' \)

where \( \bar{V} \) is value of filing for bankruptcy:

\[ \bar{V}_j(z, \eta) = u(c) - \chi + \beta E \max \{ V_{j+1}(0, z', \eta', \kappa'), \bar{W}_{j+1}(z', \eta', \kappa') \} \]

s.t. \( c = (1 - \gamma)\bar{e}_j z \eta \)

and \( \bar{W} \) is value of defaulting immediately following bankruptcy:

\[ \bar{W}_j(z, \eta, \kappa) = u(\bar{c}) - \chi + \beta E \max \{ V_{j+1}(d'(\kappa), z', \eta', \kappa'), \bar{V}_{j+1}(z', \eta', \kappa) \} \]

s.t. \( c = (1 - \gamma)\bar{e}_j z \eta, \quad d' = (\kappa - \gamma \bar{e}_j z \eta)(1 + r^r) \)
Given risk-free bond prices \((q^s, \overline{q^b})\), a recursive competitive equilibrium is value functions \(V, \overline{V}, \overline{W}\), policy functions \(c, d', I(d, z, j)\), default probabilities \(\theta(d', z, j)\), and a pricing function \(q^b\) such that:

1. Value functions satisfy functional equations, and \(c, d'\) and \(I\) are the associated optimal policy functions.
2. The bond prices \(q\) are determined by zero profit condition.
3. The default probabilities are correct:
   \[
   \theta(d', z, j) = E \left( I(d' + \kappa', z', j + 1) \right)
   \]
Methodology

- Calibrate benchmark economy to match late 90’s. **Targets:** Filings, unsecured debt, interest rates, charge-off rate.

- Run “backward” experiments trying to match early 80’s.

- Consider each story individually.
  - Changes required to match the early 80’s.
  - Plausible changes in parameters.

- Can a combination of stories match the data?
Benchmark Parametrization

- 16 periods (3 years each).
- Last period is retirement (= no shocks).
- \( u(c) = \frac{1}{1-\sigma}[c^{1-\sigma} - 1] \)
- \( \sigma = 2, \beta = 0.94^3. \)
- Interest rate on savings \( r_s = 3.44\% \).
  (average return on municipal bonds)
Parameterization: Shocks

Expense Shocks

- Use data on:
  1. Medical bills (MEPS 1996-97)
  2. Divorce (US Vital Statistics, Equivalence Scale)

- Combine to construct two expense shocks:
  1. 82% of avg. earnings with probability 0.46%
  2. 26% of avg. earnings with probability 6%

Income Shocks

- From the literature
Bankruptcy Parameters

- No stigma, $\chi = 0$
- $r^r = 20\%$
- Remaining 3 parameters ($\bar{r}$, $\tau$, $\gamma$) are set to match:
  
<table>
<thead>
<tr>
<th>Fact</th>
<th>1995-99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chapter 7 filings</td>
<td>0.83%</td>
</tr>
<tr>
<td>Average borrowing interest rate</td>
<td>11.7-13.1%</td>
</tr>
<tr>
<td>Unsecured Debt/Income ratio</td>
<td>9%</td>
</tr>
<tr>
<td>Charge-off rate</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

Transaction cost of borrowing: $\tau = 2.56\%$
Linear garnishment $\gamma = 0.319\%$
Interest ceiling, $\bar{r} = 75\%$

Interpretation of $\gamma$. (also, lower $\gamma$ would imply more defaults, less debt, and much higher interest rates.)
## Benchmark Results: Cause of Bankruptcy

<table>
<thead>
<tr>
<th>Income Shock</th>
<th>Small Exp.</th>
<th>Large Exp.</th>
<th>No Exp.</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>None</td>
<td>48.32%</td>
<td>7.93%</td>
<td>13.50%</td>
<td>69.75%</td>
</tr>
<tr>
<td>Bad Persist.</td>
<td>11.01%</td>
<td>2.22%</td>
<td>6.95%</td>
<td>20.18%</td>
</tr>
<tr>
<td>Trans.</td>
<td>5.35%</td>
<td>0.90%</td>
<td>1.53%</td>
<td>7.78%</td>
</tr>
<tr>
<td>Pers + trans.</td>
<td>1.23%</td>
<td>0.25%</td>
<td>0.80%</td>
<td>2.28%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>65.91%</strong></td>
<td><strong>11.31%</strong></td>
<td><strong>22.78%</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>
Experiments

Can Stories Work Alone?

1. Change in variance of income
   (a) Transitory
   (b) Persistent
2. Increasing expense shocks
3. Decreasing stigma
4. Decline in transaction cost of lending
5. Change in usury laws

Combining the Stories
Stigma, lending cost, expense shock, and income volatility
Experiment 1: Income Shocks

- Variance of shocks has increased
  HSV (2004): $\sigma^2_{\eta}$ up 25%, $\sigma^2_{\epsilon}$ up 42%

- Persistence of income has decreased

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt earnings</th>
<th>avg. $r^b$</th>
<th>charge-off</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995-99 (model/data)</td>
<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
<td>4.9%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
<td>1.9%</td>
</tr>
<tr>
<td>$\sigma^2_{\eta}$ ↓, $\sigma^2_{\epsilon}$ ↓</td>
<td>0.822%</td>
<td>12.1%</td>
<td>9.8%</td>
<td>3%</td>
</tr>
<tr>
<td>$\sigma_{\eta} = 0$</td>
<td>0.83%</td>
<td>12.25%</td>
<td>8.83%</td>
<td>2.7%</td>
</tr>
<tr>
<td>$\sigma_{\epsilon} = 0$</td>
<td>0.68%</td>
<td>27.5%</td>
<td>6.99%</td>
<td>1%</td>
</tr>
</tbody>
</table>

Conclusion: Cannot generate large change in filings.
# Experiment 2: Expense Shocks

**Aim:** Decrease expense shocks to match 1980-84 filings

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt/earnings</th>
<th>avg. $r^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995-99 (model/data)</td>
<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
</tr>
<tr>
<td>No small shock</td>
<td>0.25%</td>
<td>8.91%</td>
<td>8.6%</td>
</tr>
<tr>
<td>No large shock</td>
<td>0.74%</td>
<td>8.89%</td>
<td>11.5%</td>
</tr>
</tbody>
</table>

## Conclusion:
- Extreme changes in expense shocks can match filings.
- But generates insufficient changes in debt/income ratio.

What is a realistic change in expense shocks?
Experiment 2.2: Realistic Expense Shocks

Increase in Out-Of-Pocket Medical Spending in the Data

- Real OOPS per HH: $1,477 in 1980 → $1,946 in 1998.
- As fraction of median income: 3.55% → 4.16%.
- Fraction of uninsured HHs: 13% in 1987 → 16% in 1998.

Experiment: Decrease magnitudes and probabilities by 15%.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt/earnings</th>
<th>avg. $r^b$</th>
</tr>
</thead>
<tbody>
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<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
</tr>
<tr>
<td>15% decrease</td>
<td>0.73%</td>
<td>9.03%</td>
<td>10.9%</td>
</tr>
</tbody>
</table>

The probability of family-related shocks has gone down, not up!
Experiment 3: Stigma

Aim: Introduce stigma to match filings in 1980-84. Achieved with utility loss $\approx$ consumption loss of 28%.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt/earnings</th>
<th>avg. $r^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995-99 (model/data)</td>
<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
</tr>
<tr>
<td>Stigma</td>
<td>0.25%</td>
<td>12.89%</td>
<td>7.9%</td>
</tr>
</tbody>
</table>

Conclusion: Can match the change in filings rates **but** generates counterfactual debt/income and interest rates. Robustness: get very similar results with non-utility costs.
Decline in Bankruptcy Cost – Interpretation

- Changes in social norms – reduced stigma (Fay, Hurst and White 2002).
- Reduced cost of accessing credit after bankruptcy (Staten 1993).
## Experiment 4: Transaction Cost

**Experiment:** Increase transaction cost $\tau$ (benchmark = 2.56%).

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt/earnings</th>
<th>avg. $r^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995-99 (model/data)</td>
<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
</tr>
<tr>
<td>$\tau = 4.81%$</td>
<td>0.79%</td>
<td>6.00%</td>
<td>15.89%</td>
</tr>
<tr>
<td>$\tau = 5.81%$</td>
<td>0.78%</td>
<td>5.00%</td>
<td>17.97%</td>
</tr>
<tr>
<td>$\tau = 6.81%$</td>
<td>0.77%</td>
<td>4.22%</td>
<td>20.08%</td>
</tr>
</tbody>
</table>

**Conclusion:**
- Small effect on filings.
- Too large change in average interest rate.
Is a Large Fall in $\tau$ reasonable?

- Lender’s zero-profit condition: charge-off rate $= \frac{r-(r^s+\tau)}{1+r}$.
- Using data: $\tau$ falls roughly from 6 to 3, a large decline.
- Redo calculations with decline in average marginal tax rate from 24.7% to 0%: much smaller decline in $\tau$.
- Stango (1999): 60% of tax filers do not itemize (no tax deduction). Also, lower income households pay lower taxes, so deduction is lower.
- We also want to interpret $\tau$ more broadly, capturing other borrowing costs that would not show up as wedge in data (e.g. fixed cost of obtaining a loan).
- Clearly more work is needed here.
Experiment 5: Usury Law

1978 *Marquette* Decision essentially removed any interest caps.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt/earnings</th>
<th>avg. $r^b$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
</tr>
<tr>
<td>$\bar{r} = 10%$</td>
<td>0.68%</td>
<td>8.9%</td>
<td>8.25%</td>
</tr>
<tr>
<td>$\bar{r} = 8%$</td>
<td>0.59%</td>
<td>2.04%</td>
<td>7.79%</td>
</tr>
</tbody>
</table>

**Conclusion:**
- Tight interest rate ceiling affects filing rates.
- Implies large changes in debt and interest rates.
- No comparable change in law in Canada.
## Experiment 6: Combination

Combine Stigma, Transactions Costs, Income and Expense

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Defaults</th>
<th>Debt earnings</th>
<th>avg. $r^b$</th>
<th>charge-off</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995-99 (Model/Data)</td>
<td>0.84%</td>
<td>9.04%</td>
<td>11.7%</td>
<td>4.8</td>
</tr>
<tr>
<td>1980-84 Data</td>
<td>0.25%</td>
<td>5.0%</td>
<td>11.6%</td>
<td>1.9</td>
</tr>
<tr>
<td>Combo</td>
<td>0.25%</td>
<td>5.24%</td>
<td>11.77%</td>
<td>1.4</td>
</tr>
<tr>
<td>No $\Delta$ Exp.</td>
<td>0.31%</td>
<td>5.21%</td>
<td>11.94%</td>
<td>1.5</td>
</tr>
<tr>
<td>No $\Delta$ Stigma</td>
<td>0.71%</td>
<td>4.35%</td>
<td>18.18%</td>
<td>6.1</td>
</tr>
<tr>
<td>No $\Delta$ $\tau$</td>
<td>0.31%</td>
<td>12.74%</td>
<td>7.93%</td>
<td>1.0</td>
</tr>
<tr>
<td>No $\Delta$ Transitory</td>
<td>0.27%</td>
<td>5.25%</td>
<td>11.82%</td>
<td>1.4</td>
</tr>
</tbody>
</table>

**Conclusion:**

- The combination of stories accounts for the rise.
- Stigma and transaction cost are most important.
Welfare Implications of Rise in Bankruptcies

Welfare measure: Equivalent Consumption Variation
Compare preferred combo with benchmark: early 1980s to late 1990s.

<table>
<thead>
<tr>
<th>comparison</th>
<th>ECV</th>
</tr>
</thead>
<tbody>
<tr>
<td>full</td>
<td>+ 0.57%</td>
</tr>
<tr>
<td>only $\tau \downarrow$</td>
<td>+ 1.19%</td>
</tr>
<tr>
<td>only $\chi \downarrow$</td>
<td>+ 0.27%</td>
</tr>
<tr>
<td>$\tau$ and $\chi \downarrow$</td>
<td>+ 1.17%</td>
</tr>
<tr>
<td>only expense risk $\uparrow$</td>
<td>– 0.29%</td>
</tr>
<tr>
<td>only earnings risk $\downarrow$</td>
<td>– 0.33%</td>
</tr>
</tbody>
</table>
Savings in Model vs. Data

- Stylized fact: decline in household savings rate.
- Model has little to say about aggregate savings rate (always zero, because no growth).
- Look at net worth instead.

<table>
<thead>
<tr>
<th>median net worth/median income</th>
<th>1984</th>
<th>1998</th>
<th>% change</th>
</tr>
</thead>
<tbody>
<tr>
<td>data</td>
<td>1.24</td>
<td>0.89</td>
<td>28% fall</td>
</tr>
<tr>
<td>model</td>
<td>0.60</td>
<td>0.40</td>
<td>34% fall</td>
</tr>
</tbody>
</table>

Note: no housing, no bequest motive in model.
Savings over the Life Cycle

Saving Rate over Life Cycle

Rise in Bankruptcies – p. 34/41
Conclusion

- No single story can account for all the key facts.
- Combination of stories can account for all the key facts.
- Two main forces:
  - Decrease in stigma,
  - Decrease in transaction cost of borrowing.
- Changes in uncertainty play small role quantitatively.
- Demographic changes are quantitatively unimportant.

We view $\tau \downarrow$ and $\chi \downarrow$ as reduced form ways of modeling technological progress in financial sector.

Current work: better understanding of financial innovation (credit scoring = better information processing).
## Summary of Experiments

<table>
<thead>
<tr>
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</tr>
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<td>11.7%</td>
</tr>
<tr>
<td>1980-84 data</td>
<td>0.25%</td>
<td>5%</td>
<td>11.5%</td>
</tr>
<tr>
<td>Realistic Income</td>
<td>0.822%</td>
<td>12.1%</td>
<td>9.8%</td>
</tr>
<tr>
<td>No Transitory</td>
<td>0.818%</td>
<td>11.7%</td>
<td>9.4%</td>
</tr>
<tr>
<td>No Persistent</td>
<td>0.63%</td>
<td>20.6%</td>
<td>8.01%</td>
</tr>
<tr>
<td>Realistic Expense</td>
<td>0.73%</td>
<td>9.03%</td>
<td>10.9%</td>
</tr>
<tr>
<td>No small shock</td>
<td>0.25%</td>
<td>8.91%</td>
<td>8.6%</td>
</tr>
<tr>
<td>Stigma</td>
<td>0.25%</td>
<td>12.89%</td>
<td>7.9%</td>
</tr>
<tr>
<td>Transaction Cost</td>
<td>0.81%</td>
<td>4.06%</td>
<td>20.16%</td>
</tr>
<tr>
<td>Usury $\bar{r} = 8%$</td>
<td>0.59%</td>
<td>2.04%</td>
<td>7.79%</td>
</tr>
<tr>
<td>Combination</td>
<td>0.25%</td>
<td>5.24%</td>
<td>11.77%</td>
</tr>
</tbody>
</table>
Implied Bankruptcy Rates (per 1,000 25+ adults), U.S.
(holding marital status specific filing rates constant)

At 1991 filing rates

Actual
constructed bankruptcy rates per 1,000 households (u.s.)
(holding age specific filings rates constant)

at 1991 filing rates
at 2001 filing rates
actual
“Family” Expense Shocks

The probability of family related shocks has gone down, not up!

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Births per 1,000 population</td>
<td>15.9</td>
<td>14.3</td>
</tr>
<tr>
<td>Births per 1,000 women aged 15-44</td>
<td>68.4</td>
<td>64.3</td>
</tr>
<tr>
<td>Births per 1,000 unmarried women</td>
<td>29.4</td>
<td>43.3</td>
</tr>
<tr>
<td>Intended Births</td>
<td>61.9%</td>
<td>69%</td>
</tr>
<tr>
<td>Births per 1,000 teenagers (15-19 yrs old)</td>
<td>53.0</td>
<td>50.3</td>
</tr>
<tr>
<td>Divorces per 1,000 population</td>
<td>5.3</td>
<td>4.1</td>
</tr>
</tbody>
</table>
Labor Income Process

- Age profile of earnings (Gourinches and Parker (2002))
- 5 persistent productivity shock values: 
  \[ z \in \{ z_1, z_2, z_3, z_4, z_5 \} \]
  Tauchen method to discretize AR(1).
  \[
  \log z^i_j = \rho \log z^i_{j-1} + \epsilon^i_j
  \]
  where \( \rho = 0.96, \sigma^2_{\epsilon} = 0.014 \).
- 3 transitory shock values: \( \eta \in \{ \eta_1, 1, \eta_3 \} \)
  \( \sigma^2_{\eta} = 0.05 \).
  Support: \( \pi_1 = \pi_3 = 0.1 \)