Efficiency with Endogenous Population Growth

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A Quote from The Economist

”Some people fret that if more women work rather than mind their children, this will boost GDP but create negative social externalities, such as a lower birth rate. Yet developed countries where more women work, such as Sweden and America, actually have higher birth rates than Japan and Italy, where women stay at home.”

Women in the Workforce, April 12 2006

Motivation

- Should fertility be limited by law? (China)
- Should fertility be ‘discouraged’? (UK, Canada, etc.)
  - Family planning services (birth control/abortions)
- Should fertility be ‘encouraged’?
  - Estonia pays mothers full salary for up to 1 year.
  - France spends 4.5% of GDP on family policy.
  - Japan organizes group dates to spur marriages and babies.
- Are ‘births’ an externality?
- Is fertility ‘wrong’ in these countries so that government intervention is needed?

Research Question

1. What is the appropriate extension of Pareto optimality when fertility is endogenous?
2. Given an extended notions of Pareto optimality, under what conditions is the First Welfare Theorem valid?

1. Multiple 'reasonable' ways of generalizing PO

An Example:

- One person alive at time 0.
- Fertility choices: 1 child, 2 children, no children.
- Fertility choice determines number of people alive at time 1.
- Who gets to 'vote' in a Pareto comparison?
  - Only the parent?
  - All three? But what is the utility of an 'unborn' person? Debate in philosophy whether utility of unborn (unconceived) can be defined (e.g. Bayles 1976, Kavka 1982).
- Thus, we will define two concepts.

2. Problems for the First Welfare Theorem

- Altruism in the Family
  - Parents care about kids.
  - Kids care about parents.
  - Kids care about siblings, etc.
- Parents' decisions about fertility affect the choice sets of their potential offsprings (consumption set externality).
  $\rightarrow$ no children implies children will have no children, etc.

Summary of Results

- Two definitions of PO: $\mathcal{P}$-efficiency and $\mathcal{A}$-efficiency
- FWT's for each definition.
  - One strong new assumption is required: Maximization in the dynasty
- Some non-cooperative foundations for dynastic maximization.
- Examples illustrating why the FWT might fail.
- With global externalities (e.g. pollution): Pigouvian taxes may not be enough, also need fertility tax!

Literature

- Long-standing debate on optimal population size:
  Malthus (1798), Bentham (1823), Mill (1848), etc.
- Informal discussion of externalities in the fertility context:
  e.g. Chomitz and Birdsall (1991)
- Use of social welfare functions:
  Nerlove, Razin and Sadka (1987, 1989), Razin and Sadka (1995)
- Social Choice Literature: Axiomatic Approach (Blackorby, Bossert and Donaldson 1995, 2002).
- Schweizer (1996), Michel and Wigniolle (2003),
  Conde-Ruiz et al (2004) are similar to our approach, but less general.

The Environment

- Overlapping generations economy
- Agents consume during one period only.
- Initial population: $\mathcal{P}_0 = \{1, \ldots, N\}$
- Each person can give birth to maximal $\bar{f}$ children.
- Define *potential population* recursively $\mathcal{P}_t \equiv \mathcal{P}_{t-1} \times \{1, \ldots, \bar{f}\}$.
- Example of a (potential) person: $i = (1, 2, 1)$.
- Fertility of i is denoted by $f(i) \in [0, \bar{f}]$.
- $I(f) \subseteq \mathcal{P}$ – set of people born under fertility plan f.
- $e(i)$ endowment of person i if born.
- $c(f(i)) \in \mathbb{R}^k$ cost of child rearing

Our Approach: Explicit Dynastic Structure

Why?

- Allows for external effects between family members.
- Makes it explicit that “adding” a person is costly, and cost might not be transferable.
- Introduces a natural asymmetry between the initial generation and future people.

Allocations and Preferences

- k goods
- $x(i) \in \mathbb{R}^k_+$ is consumption of person i
- An allocation is $(x, f) = (x(i), f(i))_{\{i \in I(f)\}}$.
- Agents have preferences over consumption and fertility.
- $u_i(x, f) = u_i(x(i), f(i), x(-i), f(-i))$

Assumption 1 *for each $i \in \mathcal{P}$, there is a well defined, real-valued utility function $u_i : A \to \mathbb{R}$, where A is the set of all allocations.*

Assumption 2 *for each $i \in \mathcal{P}$, there is a well defined, real-valued utility function $u_i : A(i) \to \mathbb{R}$, where $A(i)$ is the set of all allocations in which i is born.*

Feasibility

Definition 1 *An allocation* (f, x) *is feasible if*

1. $(f(i), x(i)) \in Z$, *for all* $i \in I(f)$,
2. $\sum_{i \in I_t(f)} x(i) + \sum_{i \in I_t(f)} c(f(i)) = \sum_{i \in I_t(f)} e(i)$ *for all* t,

(In paper: all results carry through to production economy.)

$\mathcal{P}$-Efficiency

Assume Assumption 1 holds.

Definition 2 *A feasible allocation $z = \{(x_i, f_i)\}_i$ is $\mathcal{P}$-efficient if there is no other feasible allocation $\hat{z}$ such that*

1. $u_i(\hat{x}_i, \hat{f}_i, \hat{z}_{-i}) \geq u_i(x_i, f_i, z_{-i})$ *for all* $i \in \mathcal{P}$
2. $u_i(\hat{x}_i, \hat{f}_i, \hat{z}_{-i}) > u_i(x_i, f_i, z_{-i})$ *for at least one* $i \in \mathcal{P}$.

Proposition 1 *Assume for all i, $u_i(z) = u_i(z')$ for all z, z' in which i is not alive. Then, if an allocation is $\mathcal{P}$-efficient, it is Pareto Optimal among the alive agents.*

Inefficiently high fertility?

Proposition 2 *If the allocation* (f^*, x^*) *satisfies* $u_i(z^*) > \bar{u}_i(unborn)$ *for all* $i \in I(z^*)$*, and if the allocation* (f', x') *is* $\mathcal{P}$*-Superior to* (f^*, x^*)*, then* $I(f^*) \subseteq I(f')$*.*

$\mathcal{A}$-Efficiency

Assume Assumption 2 holds

Definition 3 *A feasible allocation* $z = \{(x_i, f_i)\}_i$ *is* $\mathcal{A}$*-efficient if there is no other feasible allocation* $\hat{z}$ *such that*

1. $u_i(\hat{f}, \hat{x}) \geq u_i(f, x) \;\; \forall i \in I(f) \cap I(\hat{f})$
2. $u_i(\hat{f}, \hat{x}) > u_i(f, x))$ *for some* $i \in I(f) \cap I(\hat{f})$

Advantage: No need to define $u_i(unborn)$, i.e. do not need Assumption 1.
Disadvantage: May not exist (generically it does exist), because notion of $\mathcal{A}$-dominance is not transitive.

Example 1

- 2-period example, one parent.
- Endowments: e_0 and e_1.
- No production, no storage
- e_1 not transferable across children.
- Cost of child-rearing: $\theta > 0$. Assume $e_0 > \theta \bar{f}$.
- Parent: $u_1(c(1), f(1); c(1,1), ..., c(1, f(1))) =$

$$\begin{cases} u(c(1)) + \beta \frac{1}{f(1)^\eta} \sum_{j=1}^{f(1)} u(c(1,j)), \text{ if } f(1) > 0 \\ u(c(1)), \text{ if } f(1) = 0 \end{cases}$$

- Children: $u(c(1,i))$ if born, $\bar{u}$ otherwise.

Example 1 continued

- Define

$$W(f) \;=\; u(e_0 - \theta f) + \beta f^{1-\eta} u(e_1),$$

- Assume $W(f)$ has a unique maximizer, call it f^*.
- f^* is $\mathcal{A}$-efficient.
- No other allocation is $\mathcal{A}$-efficient.
- Case 1: $u(e_1) > \bar{u}$, then any $f \in \{f^*, \ldots, \bar{f}\}$ is $\mathcal{P}$-efficient. Any lower fertility can be $\mathcal{P}$-dominated.
- Case 2: $u(e_1) < \bar{u}$, then any $f \in \{0, \ldots, f^*\}$ is $\mathcal{P}$-efficient.

Example 2 – add storage and transferability

- Feasibility:

$$c(1)+f(1)\theta+\sum_{j=1}^{f(1)} c(1,j) \leq e_0+f(1)e_1 \text{ and } c(1) \leq e_0-f(1)\theta$$

- Again, the allocation that is best for the parent is $\mathcal{A}$-efficient, but there are many others.
- Let $e_0 = 100, e_1 = 0, \theta = 24, \beta = 1, \eta = 0$ and $u(c) = \sqrt{c}$.
- Parent's utility: $u(c(1)) + u(c(1,1)) + u(c(1,2)) + \ldots$.
- Given parameters, parent's most preferred allocation is $c(1) = c(1,1) = c(1,2) = \frac{100-48}{3}$ and is $\mathcal{A}$-efficient.
- Consider allocation with one child: $c(1) = c(1,1) = \frac{100-24}{2}$, this is also $\mathcal{A}$-efficient.

Example 2 continued

Allocation $f(1) = 1$ and $c(1) = c(1,1) = \frac{100-24}{2}$ is also $\mathcal{A}$-efficient because:

- Holding fertility constant, there is no way of improving either parent or child without hurting the other.
- Allocation with 0 children is strictly worse for the parent.
- Allocation with 2 children, but holding first child constant, is also worse for the parent.
- All allocations with 2 children that are better for the parent make the first child worse off.

Characterization Results

Result 1 *Pick* $\{a(i)\}_{i\in\mathcal{P}}$ *with* $a(i) > 0, \ \ \forall i \in \mathcal{P}$. *Suppose* (f^*, x^*) *is a solution to*

$$\max_{(f,x)} \sum_{i\in\mathcal{P}} a(i)u_i(f,x) \quad , \tag{1}$$

s.t. feasibility and suppose that $\sum_{i\in\mathcal{P}} a(i)u_i(f^*,x^*) < \infty$. *Then* (f^*, x^*) *is* $\mathcal{P}$*-efficient.*

Result 2 *Pick* $\{a(i)\}_{i\in\mathcal{P}_0}$ *with* $a(i) \geq 0 \ \ \forall i \in \mathcal{P}_0$. *Suppose* (f^*, x^*) *is the* <u>*unique*</u> *solution to:*

$$\max_{(f,x)} \sum_{i\in\mathcal{P}_0} a(i)u_i(f,x) \quad , \tag{2}$$

s.t. feasibility and suppose that $\sum_{i\in\mathcal{P}_0} a(i)u_i(f^*,x^*) < \infty$. *Then* (f^*, x^*) *is* $\mathcal{A}$*-efficient.*

Relationship between concepts

1. Critical Level Utilitarianism (Blackorby et al)

$$W(f, x; \alpha) = \sum_{i \in I(f)} [u_i(f, x) - \alpha] \tag{3}$$

   - α is interpreted as an ethical parameter.
   - If $\bar{u}_i = \alpha$ for all i, then maximizer of $W(f, x; \alpha)$ (subject to feasibility) is $\mathcal{P}$-efficient.
   - Can find α such that maximizer of $W(f, x; \alpha)$ is $\mathcal{A}$-efficient, but may need α_i or α_t.

2. With a few additional assumptions, can show that $\mathbb{A} \subseteq \mathbb{P}$.

Decentralized Fertility Decisions

When do decentralized fertility decisions lead to efficient allocations?

Problem: externalities (family altrusim)

Divide the problem into 2 steps:

1. Assume externalities confined to dynasties (D_i).
   Assume dynasties solve dynasty problem efficiently.
   $\longrightarrow$ Focus on inter-family interaction.
2. When does individual maximizing behavior lead to dynastically efficient outcomes?
   $\longrightarrow$ Focus on dynamic games between family members.

Step 1: Dynastic Optimization

Definition 4 *Given p, a dynastic allocation for dynasty i, $(f_i, x_i) = \{f(j), x(j)\}_{j \in D_i}$ is said to be Dynastically $\mathcal{P}$-maximizing if $(f(j), x(j)) \in Z$ for all $j \in I(f_i)$ and $\sum_t p_t \sum_{j \in \mathcal{P}_t \cap I(f_i)} (x(j) + c(f(j))) \leq \sum_t p_t \sum_{j \in \mathcal{P}_t \cap I(f_i)} e(j)$, and if $\nexists (\hat{f}_i, \hat{x}_i) = \{\hat{f}(j), \hat{x}(j)\}_{j \in D_i}$ such that:*

1. $(\hat{f}(j), \hat{x}(j)) \in Z$ *for all* $j \in I(\hat{f}_i)$.
2. $u_j(\hat{f}_i, \hat{x}_i) \geq u_j(f_i, x_i)$ *for all* $j \in D_i$.
3. $u_j(\hat{f}_i, \hat{x}_i) > u_j(f_i, x_i))$ *for at least one* $j \in D_i$.
4. $\sum_t p_t \sum_{j \in \mathcal{P}_t \cap I(\hat{f}_i)} (\hat{x}(j) + c(\hat{f}(j))) \leq \sum_t p_t \sum_{j \in \mathcal{P}_t \cap I(\hat{f}_i)} e(j)$.

First Welfare Theorem

Definition 5 *(p^*, f^*, x^*) is a dynastic $\mathcal{P}$-equilibrium if*

1. *For all dynasties i, given p^*, (f_i^*, x_i^*) is dynastically $\mathcal{P}$-maximizing.*
2. *(f^*, x^*) is feasible.*

A Dynastic $\mathcal{A}$-equilibrium is defined similarly.

Assumption 3 *No externalities across dynasties.*

Proposition 3 *Assume preferences are strictly monotone in (x_i, f_i) for all $i \in \mathcal{P}_0$. Let (p^*, x^*, f^*) be a dynastic $\mathcal{P}(\mathcal{A})$-equilibrium, then (x^*, f^*) is $\mathcal{P}(\mathcal{A})$-efficient.*

$\Rightarrow$ To the extent that dynasties maximize, equilibrium fertility decisions are efficient.

Step 2

When does individual optimizing behavior lead to outcomes that are dynastically maximizing?

- Equilibrium concept: Subgame perfect Nash among family members, embedded in a standard Walrasian equilibrium.
- No reason to believe outcomes should be efficient. (external effects)
- But if family members preferences are similar enough or if bequest space is rich enough, then equilibria are efficient.
  - “Perfect altruism” (Barro Becker model)
  - Rich enough set of available contracts (e.g. bequests conditional on not smoking)

Time Consistent Preferences

Barro and Becker (1988,1989)

- "Perfect altruism" within dynasty.
- Generalize to multiple dynasties.
- No external effects across dynasties.
- Allow asymmetric treatment of children.
- Preferences:

$$U_t(i^t) = u(x_t(i^t)) + \beta g(f_t(i^t)) \int_0^{f_t(i^t)} U_{t+1}(i^{t+1}) di_{t+1}$$

- Strategies: $(x, f, b(\cdot))$, where $b(\cdot)$ is bequest, can be negative!

Theorem

If

- *U is continuous and $U(0) = 0$, and,*
- *$c(f) = \theta f$, and,*
- *$g(xy) = g(x)g(y)$, and,*
- *$Fg(F)U(X/F)$ is strictly concave in (F, X).*

Then,

- *For every T and every p, the T period truncated game has a unique subgame perfect equilibrium outcome.*
- The limit of this SPE outcome as $t \to \infty$ is a SPE outcome of the infinite horizon game.
- *This SPE outcome is* $\mathcal{P}$- and $\mathcal{A}$-efficient.

Example: Kids and Drugs

- $\mathcal{P} = \{1, (1,1)\}$
- Parent: $u_1 = u(c_1) + f_1 \beta u(c_{(1,1)})$
- Child: $u_{(1,1)} = f_1[c_{(1,1)} + \gamma d_{(1,1)}]$, $\gamma > 1$
- Budget constraint period 0: $c_1 + f_1 \theta \leq w$
- Budget constraint period 1: $c_{(1,1)} + d_{(1,1)} \leq f_1 w$.
- Equilibrium allocation:
  $z = \{c_1 = w, f_1 = 0, c_{(1,1)} = 0, d_{(1,1)} = 0\}$
- But $\hat{z} = \{c_1 = w - \theta, f_1 = 1, c_{(1,1)} = w, d_{(1,1)} = 0\}$ is $\mathcal{P}$-superior.

Fertility can be inefficiently high/low because...

1. of the usual reason: External effects across dynasties.
2. family games can lead to outcomes that are not optimal for dynasty as a whole (e.g. drugs example).

Fertility & Land Scarcity

- Will fertility be inefficiently high with scarce factors?
- Logic: Parents do not take into account that more children decrease land available per capita.
- $w_t = F_L(\bar{A}, N_t)$ decreases in N_t.
- Is this an externality?
- No! It's analogue to the effect an individual's increase in labor supply has on aggregate labor and thereby wages.
- So all assumptions of our 1st W.T. are satisfied.
  $\rightarrow$ No inefficiency.

Pollution, Pigouvian Taxes, and Fertility

A 2-period Example

1. Parents (period 1):

$$\max u(c) + \beta f^{\alpha}[fV]$$
$$s.t. c_1 + (\theta + \tau_f)f \leq 1 + T_1$$

2. Children (period 2):

$$V = \max v(c_2, C) - l$$
$$(1 + \tau_c)c_2 = l + T_2$$

3. Pollution: $C = fc_2$

Equilibrium vs. Optimum

Equilibrium FOC:

$$u'(c_1)(\theta + \tau_f) = \beta(\alpha + 1) f^{\alpha}[v(c_2, fc_2) - c_2] \tag{4}$$

$$v_1 = 1 + \tau_c \tag{5}$$

Symmetric $\mathcal{A}$-Planner:

$$u'(c_1)\theta = \beta(\alpha + 1) f^{\alpha}[v(c_2, fc_2) - c_2] + \beta f^{\alpha+1} v_2 c_2 \tag{6}$$

$$v_1 = 1 - f v_2 \tag{7}$$

Observations:

- Set $\tau_f = 0$. There is no τ_c such that (4) and (6) both hold.
- Pigouvian tax $\tau_c = -f v_2$ corrects overproduction in period 2, but not additional pollution caused by too many people.
- $\tau_f = -f^{\alpha+1} c_2 v_s / u'(c_1)$ leads to an $\mathcal{A}$-superior allocation.

Conclusion/Summary

- Two definitions of PO: $\mathcal{P}$-efficiency and $\mathcal{A}$-efficiency
- FWT's for each definition.
  - One strong new assumption is required: Maximization in the dynasty
- Some non-cooperative foundations for dynasty maximization.
- Equilibria can be inefficient if families aren't 'cooperative' enough.
- Endogenous fertility exacerbates 'usual' externalities.

Figure 2

Assumption: $u(e_1) > \bar{u}$