Families as Roommates

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November 2007
Motivation

1. Large decrease in household size over last 150 years. → What can explain this decline?

2. Typical analysis: concentrate on specific change in living arrangements:
   - increasing marriage age
   - decreasing fertility
   - increasing divorce rates
   - decline of extended family

3. We believe that these are different manifestations of the same phenomenon: people can afford to live in smaller households.

4. Important for policy analysis: decline in family size not necessarily a concern, but simply an efficient response to growing incomes.
Outline of the Talk

1. Some facts: Changes in household size from U.S. Census data.
2. A model of household size choice.
3. We use 1995-2000 data to calibrate the model.
5. Result: increase in income can account for about 30% of the observed decline in household size.
6. Adding Children: model can account for entire HH size decline.
Various Measures of Household Size
(excluding group quarters)
average across all people

Year

Number
0 1 2 3 4 5 6 7 8

children
adults
Hhsize
famsize
Number in Household of Average Person

Relationship to Head

1880
2000
Household Size by Birth Cohorts

- 1820–1840
- 1860–1880
- 1900–1920
- 1940–1960

Age

Household Size

0 10 20 30 40 50 60 70 80 90 100

0 1 2 3 4 5 6 7 8
### Average Household Size by Income Quintiles, 2000, 30-34 year old persons

<table>
<thead>
<tr>
<th>Q</th>
<th>kids</th>
<th>adults</th>
<th>total</th>
<th>non-family</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.69</td>
<td>2.44</td>
<td>4.13</td>
<td>0.32</td>
</tr>
<tr>
<td>2</td>
<td>1.55</td>
<td>2.31</td>
<td>3.86</td>
<td>0.23</td>
</tr>
<tr>
<td>3</td>
<td>1.40</td>
<td>2.14</td>
<td>3.54</td>
<td>0.16</td>
</tr>
<tr>
<td>4</td>
<td>1.17</td>
<td>1.99</td>
<td>3.17</td>
<td>0.11</td>
</tr>
<tr>
<td>5</td>
<td>0.96</td>
<td>1.92</td>
<td>2.88</td>
<td>0.09</td>
</tr>
</tbody>
</table>
Summary of the Data


2. Decrease has occurred at all points in the life-cycle.

3. This is not simply a decrease in fertility.

4. Decline in all types of household members.

5. Richer people live in smaller households.
Our Story: Substitution from HH public to private goods

- People consume two types of goods:
  - household public goods (living room, TV, garden)
  - pure private goods (dining out, plane trip, movie tickets)
- Benefit of living together: Public goods.
- Time cost of forming/maintaining a HH.
- As people get richer (GDP p.c. ↑)
  - They want to consume relatively more private goods.
  - Benefit from living together declines endogenously.
  - People choose to live in smaller households.
The Model

• Life-cycle model: \( a = \text{age} \).

• OLG: \( \tau = \text{birth cohort} \).

• Finite number of types in each cohort: \( i \).

• Efficiency units of time: \( z(\tau, a, i) \).

• Household specific public good, \( h \).

• Private good, \( v \).

• Household size, \( s \).

• Age-specific household creation/maintenance (time) costs: \( B_a z \).

• Exogenous increase in productivity \( z \) over time.
Problem of a Consumer of type \((\tau, i)\)

\[
\begin{align*}
\max_{s,v,h} & \quad \sum_{a=0}^{\bar{a}} \beta^a \left[ \frac{h(a)^{1-\sigma}}{1-\sigma} + \omega \frac{v(a)^{1-\phi}}{1-\phi} \right] \\
\text{s.t.} & \quad \sum_{a=0}^{\bar{a}} p(\tau + a) \left[ \frac{h(a)}{s(a)} + v(a) \right] \\
& \quad \leq \sum_{a=0}^{\bar{a}} p(\tau + a)[1 - B_a(s(a) - 1)]z(\tau, a, i) \\
& \quad s(a) \geq 1 \quad \forall a \\
& \quad v(a), h(a) > 0 \quad \forall a
\end{align*}
\]

Notation: \(s(\tau, a, i)\) is optimal household size of agent born in \(\tau\) of age \(a\) and type \(i\).
Families as Roommates

• This is not a dynamic theory of family formation.

• There is no cost of changing HH size from one period to the next (e.g. no cost to get divorced).

• Instead: every period people can choose who to live with.

• Household members = roommates who share the costs of the public goods, but impose a cost of living together on each other (e.g. time spend arguing about who washes the dishes).

• Too simple?

• Well, let’s see how far one get get with such a simple theory...
Equilibrium

An equilibrium for this economy is an allocation \( \{s(\tau, a, i), v(\tau, a, i), h(\tau, a, i)\}_{\tau, i} \) and prices \( \{p(t)\} \) such that:

1. Each agent type \((\tau, i)\) maximizes utility subject to the constraints.

2. Markets clear every period:

\[
\sum_{\{(\tau,a,i)|\tau+a=t\}} \left[ \frac{h(\tau, a, i)}{s(\tau, a, i)} + v(\tau, a, i) \right] = \sum_{\{(\tau,a,i)|\tau+a=t\}} [1 - B_a(s(\tau, a, i) - 1)]z(\tau, a, i)
\]
Household Size and Public Good Share

Result 1 \( \frac{ds}{dz} < 0 \) if and only if \( \frac{d(h/z)}{dz} < 0 \).

Proof. From the FOCs: \( \frac{h}{z} = Bs^2 \).

Household Size in the Cross-section

Result 2 Suppose that \( z(\tau, a, i) = z(\tau, i) \) for all \( a \).
Assume \( \sigma > 0.5, \sigma > \phi \). Then \( z(\tau, i) > z(\tau, j) \) implies that \( s(\tau, a, i) \leq s(\tau, a, j) \) for all \( a \), with strict equality if \( 1 < s(\tau, a, i) \).
Household Size Across Cohorts

Result 3  Suppose $B_a = B$ for all $a$ and that for all $i$, $z(\tau, a, i) = z(\tau, i)$ for all $a$.

a) If $\sigma > 0.5$, $\phi < \sigma$, then $z(\tau', i) > z(\tau, i)$ implies that $s(\tau', a, i) < s(\tau, a, i)$ for all $(a, i)$.

b) If $\sigma > 0.5$, $\phi > \sigma$, then $z(\tau', i) > z(\tau, i)$ implies that $s(\tau', a, i) > s(\tau, a, i)$ for all $(a, i)$. 
Empirical Strategy

- Our theory proposes that if $\sigma > \phi$, then higher incomes lead to a larger private goods share and smaller households.

- How do we know if $\sigma > \phi$?

- We use cross-sectional data (from CEX) to test $\sigma \geq \phi$ and to calibrate our model.

- We then project the model back to 1850 to see how important this channel is in explaining the falling household size.
Consumer Expenditure Survey

- Use 1995-2000 as a cross-section.
- Detailed expenditure data, plus income data.
- Public goods ($h$) = housing, utilities, books, house services.
  Private goods ($sv$) = food, health care, education, clothing, transport, personal services, entertainment.

- Exclude most durable goods.
CEX Data: s, v and h by Income Quintiles

- Break people into five income types in model and data.
- In data we identify these with five income quintiles.

<table>
<thead>
<tr>
<th>quintile</th>
<th>HH size</th>
<th>$h$</th>
<th>$v$</th>
<th>$h/v$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.33</td>
<td>1,600</td>
<td>534</td>
<td>3.00</td>
</tr>
<tr>
<td>2</td>
<td>3.90</td>
<td>2,046</td>
<td>741</td>
<td>2.76</td>
</tr>
<tr>
<td>3</td>
<td>3.56</td>
<td>2,360</td>
<td>929</td>
<td>2.54</td>
</tr>
<tr>
<td>4</td>
<td>3.07</td>
<td>2,658</td>
<td>1,166</td>
<td>2.28</td>
</tr>
<tr>
<td>5</td>
<td>2.32</td>
<td>3,200</td>
<td>1,757</td>
<td>1.82</td>
</tr>
</tbody>
</table>
Calibration Strategy

- Consider agents in 5-year age groups: 0-4, 5-9, ..., 75-79 (16 groups).
- 19 parameters: $\sigma, \phi, \omega, \{B_a\}$.
- 19 Moments to match.

<table>
<thead>
<tr>
<th>Moment</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average $h/v$ ratio for 40-49 year-old in 2000</td>
<td>2.48</td>
</tr>
<tr>
<td>Income elasticity of $h/v$ for 40-49 year-old in 2000</td>
<td>-0.24</td>
</tr>
<tr>
<td>Standard intertemporal elasticity of substitution</td>
<td>0.50</td>
</tr>
<tr>
<td>Household size for age groups from 2000 Census</td>
<td></td>
</tr>
</tbody>
</table>

- Elasticity is defined between the five income quintiles.
Calibration Results

- $\sigma = 1.91 > \phi = 1.66, \quad \omega = 0.057$

<table>
<thead>
<tr>
<th>age</th>
<th>0-4</th>
<th>5-9</th>
<th>10-14</th>
<th>15-19</th>
<th>20-24</th>
<th>25-29</th>
<th>30-34</th>
<th>35-39</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_a$</td>
<td>6.7%</td>
<td>6.3%</td>
<td>6.3%</td>
<td>7.0%</td>
<td>9.8%</td>
<td>10.4%</td>
<td>9.8%</td>
<td>8.8%</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>age</th>
<th>40-44</th>
<th>45-49</th>
<th>50-54</th>
<th>55-59</th>
<th>60-64</th>
<th>65-69</th>
<th>70-74</th>
<th>75-79</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_a$</td>
<td>9.1%</td>
<td>10.4%</td>
<td>12.7%</td>
<td>14.8%</td>
<td>16.0%</td>
<td>17.2%</td>
<td>18.4%</td>
<td>20.4%</td>
</tr>
</tbody>
</table>
Time Series Projection

• We match 2000 levels of household size and relative consumption.

• We match the 2000 elasticity of relative consumption with respect to income.

• Now we project the model backwards.
  – We use GDP/capita $Y_t$
  – Assume relative incomes are constant over time ($z_i$).
  – Then $z(\tau, a, i) = z_i Y_{t+a}$.
Cross section of household size over time

- 1850 Data
- 2000 Data
- 1850 Model
- 2000 Model
The Model vs. NIPA Data

![Graph showing aggregate H/V ratio over years from 1929 to 1999. The graph compares the model data (dashed line) and BEA data (solid line). The model data generally follows the BEA data but with slight variations.}]
Adding Children

1. So far, the model can explain about 20-30\% of the decline in household size.

2. We believe this channel is also relevant for decision to have children.

3. Modify the model to include children:
   - Adults care about children in household.
   - Children also consume private and public goods.
   - Richer parents → more private goods for their kids and fewer children.
   - A version of the “quantity-quality” trade-off.
Adults vs. Children in the Data: 3 Interesting Features

• Both, the number of children and the number of adults in a household has fallen over the last 150 years.

• The decline in the number of children is relatively larger.

• Asymmetry in timing: Most of the fall in child household size occurred before 1940, while most of the decline in adult HH size occurred after 1940.
Model with Children

\[
\max_{s,k,h,v,v^k} \sum_{a=0}^{\bar{a}} \beta^{\tau + a} U(a)
\]

\[
U(a) = \omega \frac{v(a)^{1-\phi}}{1-\phi} + \frac{h(a)^{1-\sigma}}{1-\sigma}
\]

\[
+ \delta k(a)\alpha \left\{ \Omega + \frac{h(a)^{1-\sigma}}{1-\sigma} + \omega \frac{(v^k(a))^{1-\phi}}{1-\phi} \right\}
\]

s.t. \[
\sum_{a=0}^{\bar{a}} p(\tau + a) \left[ \frac{h(a)}{s(a)} + v(a) + \frac{v^k(a)k(a)}{s(a)} \right] \leq \sum_{a=0}^{\bar{a}} p(\tau + a) z(\tau, a, i)[1 - B_a(s(a) - 1) - B_a^k k(a)]
\]
Empirical Strategy

• CEX does not distinguish between children’s and adult consumption → no data on $h, v$.
• Instead: pick parameters to match some time series moments.
## Calibration: Data Targets

<table>
<thead>
<tr>
<th></th>
<th>Kids</th>
<th>Adults</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1850 Household Size</strong></td>
<td>3.38</td>
<td>3.36</td>
</tr>
<tr>
<td>Fall, 1850-1940</td>
<td>-51.70%</td>
<td>-8.67%</td>
</tr>
<tr>
<td>Fall, 1940-2000</td>
<td>-34.27%</td>
<td>-27.30%</td>
</tr>
<tr>
<td>Quintile* 1, 2000</td>
<td>1.42</td>
<td>2.63</td>
</tr>
<tr>
<td>Quintile* 2, 2000</td>
<td>1.24</td>
<td>2.55</td>
</tr>
<tr>
<td>Quintile* 3, 2000</td>
<td>0.99</td>
<td>2.35</td>
</tr>
<tr>
<td>Quintile* 4, 2000</td>
<td>0.77</td>
<td>2.22</td>
</tr>
<tr>
<td>Quintile* 5, 2000</td>
<td>0.53</td>
<td>2.13</td>
</tr>
</tbody>
</table>

* among 25-29 year old adults.
Cross-Section: HH size by (per adult) income quintiles
2000, 25-29 year olds

- Kids (data)
- Adults (data)
- Kids (model)
- Adults (model)
Intuition for Asymmetry in Adults vs. Children

- Note: children are also a public good

- As incomes go up, people choose less public consumption ($h$) and more private goods ($v, v^k$).

- This makes children more costly. So $k$ falls.

- Adults share the cost of $h$ and $kv^k$.

- Initially $kv^k$ does not fall much, which makes it beneficial to have large adult households (to share $kv^k$ expenditures).

- Eventually $k$ has fallen so much that $kv^k$ falls and adult household size falls too.
Conclusion

- Data – Household size decline along many different margins: adults, children, non-family living together, different ages, ...
- This paper – Explores possibility of one common driving force behind these (seemingly unrelated) changes.
- Story
  - Income growth leads people to want to buy more private goods (health, movie tickets, restaurant meals, ...).
  - This endogenously decreases the benefits of (a) sharing a household with other adults and (b) having children.
- Model does fairly well in replicating data quantitatively.
Figure 1: Time Costs of Family Members

![Graph showing time costs of family members across different age groups.](image-url)
Calibrated Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$\alpha$</th>
<th>$\delta$</th>
<th>$\omega$</th>
<th>$\sigma$</th>
<th>$\phi$</th>
<th>$\Omega$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Value</td>
<td>0.5521</td>
<td>0.0795</td>
<td>$1.68 \times 10^{-5}$</td>
<td>9.5176</td>
<td>0.6592</td>
<td>0.0011</td>
</tr>
</tbody>
</table>
First Order Conditions

\[ v(\tau, a, i) : \quad \beta^a \omega v(\tau, a, i)^{-\phi} = \lambda(\tau, i)p(\tau + a) \]

\[ h(\tau, a, i) : \quad \beta^a h(\tau, a, i)^{-\sigma} = \frac{\lambda(\tau, i)p(\tau + a)}{s(\tau, a, i)} \]

\[ s(\tau, a, i) : \quad B_{a} z(\tau, a, i) = \frac{h(\tau, a, i)}{s(\tau, a, i)^2} \]
Relationship between $h, v$ and $s$

\[ h(\tau, a, i) = B_a \dot{z}(\tau, a, i) s^2(\tau, a, i) \]

\[ v(\tau, a, i) = \left( \frac{\omega h^\sigma(\tau, a, i)}{s(\tau, a, i)} \right)^{1/\phi} \]

\[ s(\tau, a, i) = \left( \frac{p(\tau)}{p(\tau + a) \beta^a} \right) \frac{1}{2\sigma - 1} \left( \frac{B_0 \dot{z}(\tau, 0, i)}{B_a \dot{z}(\tau, a, i)} \right)^{2\sigma - 1} \]